# The Quadrature of the Circle

GEOMETRICALLY & MATHEMATICALLY DEMONSTRATED

BY

JAMES SMITH.

[redacted]

THE

# QUADRATURE OF THE CIRCLE.

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OR

## THE TRUE RATIO

BETWEEN THE DIAMETER AND CIRCUMFERENCE

GEOMETRICALLY AND MATHEMATICALLY

DEMONSTRATED.

BY

JAMES SMITH Esq.

CHAIRMAN OF THE LIVERPOOL LOCAL MARINE BOARD, AND MEMBER OF THE MERSEY DOCKS AND HARBOUR BOARD.

"*Strike but Hear.*"

LIVERPOOL
EDWARD HOWELL CHURCH STREET
LONDON
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# To the Reader.

It is related, and no doubt truly, that from the falling of an apple, the acute mind of Newton was led into a train of thought and reflection, which resulted in his propounding the law of gravitation; and *you*, Reader, know as well as I do, that in the world's history it has often happened, that from the merest trifles most important events have sprung; aye, and that trifles, resulting in consequences of the greatest advantage to mankind, have frequently originated in quarters of the most unlikely description. For thirty-five years of my life I was actively engaged in commercial pursuits, and whatever may have been the natural bent of my mind, I certainly had not the leisure, and I have no recollection that during the whole of that period I had the least inclination, to turn my attention to Geometrical and Mathematical Science; but strangely enough, after I became a man of leisure, I was led to do so, from the very trifling circumstance of a friend, whose pursuits led him to the construction of mechanical drawings, telling me that he thought the area of a square on the side of an equilateral triangle inscribed in a circle, was equal to the area of the circle. I of course soon discovered this to be a fallacy; but from that hour I became powerless to resist the temptation of pursuing the search

after a square equivalent to a given circle; indeed, I believe I might as well have attempted to make an apple rise into space without the aid of some physical or mechanical appliance, as attempt to get rid of the spell, by which I found myself irresistibly bound. And yet, there is nothing unnatural in the charm by which I was so entangled; for, some of our ablest Geometers admit, "that a square equivalent to a given circle exists," and that "there is nothing visionary or absurd in the search after this square."* With this fact on record, and in pecuniary circumstances quite independent of the world, I could afford to smile at the scoffs, the sneers, and the rebuffs of "the learned;" (aye, and of the unlearned too, for I have had to do with both), and all their puny efforts to write me down a fool for my pains; one of my *learned* correspondents going the length of telling me, that "*a demon had obtained possession of my faculties;*" and another, *Mathematically unlearned*, telling his numerous readers that I am "*perfectly incorrigible in my delusions.*" Well then, I willingly admit, and no Geometer and Mathematician will quarrel with me as to the fact, that it was very improbable that anything in the shape of a geometrical and mathematical discovery, could spring from such a circumstance and such a quarter.

And now, Reader, I shall assume that you are a Geometer and Mathematician, and if so, you will not dispute the three following incontrovertible facts:—First: If the diameter of a circle be 4, the circumference and area of the circle are represented by the same arithmetical symbols, whatever the arithmetical value of $\pi$ may be. Second: If the circumference of a circle be 4, the diameter

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* "Orr's Circle of the Sciences;" the Mathematical Sciences; remarks on the Fourth Book of Euclid, page 127.

and area are represented by the same arithmetical symbols, whatever the arithmetical value of $\pi$ may be. Third: $\frac{\pi}{3 \times 100} = \frac{\pi}{300}$ is the algebraical expression of the arithmetical value of the *Natural sine* of an angle of 36 minutes; and on the Orthodox theory, which makes 3·14159 &c., the arithmetical value of $\pi$; practically 3·1416; $\frac{3 \cdot 1416}{300} = \cdot 010472$ is the *Natural sine* of an angle of 36 minutes, and is so given in all the Mathematical Tables of *Natural sines*. In the following Letter to Professor de Morgan, I make an application of the latter fact, and by means of it, demonstrate beyond the possibility of dispute or cavil, that the ratio of diameter to circumference in a circle is arithmetically expressible with perfect accuracy; and I have only to express a hope, that you, Reader, may not rank amongst that class of Mathematicians, recognised as our *guides and authorities* in Geometrical and Mathematical science; a class, who are so blinded by pride and prejudice, that they refuse to listen to truth when brought under their notice, and play the part of a "*secret confraternity of men, jealously guarding the mysteries of their profession;*"—a course of conduct denounced by the late worthy Consort of our most noble Sovereign, in his opening address to the "*British Association for the Advancement of Science,*" when he was President at the 29th Meeting of the Association, at Aberdeen.

It may so happen, however, that you, Reader, are a good Arithmetician, and yet, that you make no pretensions to be either a Geometer or a Mathematician. It may be, that you have never learned Algebra, (for this is by no means essential to make a good Arithmetician), and consequently, that the simple algebraical formula (if $\frac{4}{\pi} = x$, $\frac{4\pi}{x} = \pi^2$), would be to you as unintelligible as

so many symbols in Chinese or Hindostanee. But, I may tell you, that $\pi$ is adopted by Mathematicians to represent the circumference of a circle of which the diameter is one ; and I will now give you a translation of this algebraical formula in a school-boy form of simple arithmetic. By the formula, $\pi$ and $x$ are unknown quantities, and consequently, we may assume the value of $\pi$ to be any arithmetical quantity less than 4 and greater than 3, to prove the truth or falsehood of the formula. Well then, suppose we assume $25 \div 8 = 3{\cdot}125$, to be the arithmetical value of $\pi$ ; and I may tell you, that this value of $\pi$ makes 8 circumferences exactly equal to 25 diameters in every circle. Then : $4 \div 3{\cdot}125 = 1{\cdot}28 = x$, and 4 times $\pi$ divided by $x$, $= 12{\cdot}5 \div 1{\cdot}28 = 9{\cdot}765625 = \pi^2 = 3{\cdot}125^2$ ; therefore, $\sqrt{9{\cdot}765625} = 3{\cdot}125 = \pi$. Now, you may adopt any other arithmetical value of $\pi$ greater than $3{\cdot}125$ and less than $3{\cdot}2$, and as a mere Arithmetician work out the calculations, and in this way readily convince yourself that every intermediate value of $\pi$ would make the algebraical formula arithmetically inexpressible ; and I cannot help thinking, that you would at least have a suspicion as to the truth of the assertion of any Geometer or Mathematician, who should tell you that the algebraical formula was false, seeing that it can be worked out arithmetically by a particular value of $\pi$, to a plain, simple, harmonious, and exact result.*

* All numbers are divisible by the arithmetical quantity $3{\cdot}2$ without remainder, and we may assume this to be the arithmetical value of $\pi$. Then : $\frac{4}{\pi} = 1{\cdot}25 = x$, and $\frac{4\pi}{x} = \frac{12{\cdot}8}{1{\cdot}25} = 10{\cdot}24$ ; therefore, $\sqrt{10{\cdot}24} = 3{\cdot}2 =$ our assumed value of $\pi$. But, on the approximating theories of Orthodoxy, the results prove the truth of the Algebraical formula, whatever hypothetical value of $\pi$ we may adopt intermediate between $3{\cdot}125$ and $3{\cdot}2$. For example : We may assume the value of $\pi$ to be $3{\cdot}1416$, and if so, $\frac{4}{\pi} = \frac{4}{3{\cdot}1416} =$

Now, I assure you, my non-mathematical readers, that it has cost me years of thought and study to master the question at issue between me and the mathematical world ; and the labour of a correspondence that would fill many octavo volumes ; and yet, I may also assure you, that after all, there is only a very moderate knowledge of Geometry and Trigonometry requisite to comprehend this question ; and I have no hesitation in saying, that any fair Arithmetician may, with a few days' labour, master the contents of the Letter to Professor de Morgan, which follows this address, and qualify himself to give as good an opinion upon the subject as the Professor himself, or any other of our great "*Mathematical authorities.*"

I distributed a Pamphlet at the meeting of the "British Association" at Oxford, in 1860. This Pamphlet led me into a correspondence with an Eminent Mathematician, personally unknown to me even now, which, not giving the name of my correspondent, as a matter of course, I published. For this, critics have charged me with discourtesy, and I willingly give them all the advantage they can get by it. I have heard an anecdote of Jeffrey, one of the founders of the "Edinburgh Review," who, on being asked by an author to give a critique on his work, replied, that they never reviewed any but works of merit. Measured by this standard, this work was not contemptible, for it was reviewed by the "Athenæum," "Saturday Review," "Philosophical Magazine," and other Scientific periodicals, and from no solicitation on my part. Without being egotistical, I think I may fairly draw the same conclusion

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1·273236567354 &c., $= x$, and $\frac{4\pi}{x} = \frac{12·5664}{1·273236567354} = 9·869650560001$ &c. $= \pi^2$, and is true to 11 places of decimals; and would be literally and strictly true, if the quotient could be made to terminate at the eight decimal place. The mere arithmetical reader will not have to ask the Mathematician, for the reason why it does not so terminate !

from the following facts. The few friends to whom I sent copies of this work of course acknowledged their receipt ; but I sent copies to many of our leading *Scientific authorities*, (I will not expose them by mentioning names,) none of whom returned the book, and yet, not one of them had the courtesy to acknowledge the receipt of it. Mark the following contrast. I sent a copy to the late Prince Consort, and the receipt of it was acknowledged in the following terms:—

"Mr. Charles Ruland, Librarian to his Royal Highness the Prince Consort, has been commanded to acknowledge the receipt of Mr. James Smith's Work on the Quadrature of the Circle, forwarded through General Grey on the 16th instant ; and at the same time to thank the author for the valuable addition he has made to his Royal Highness's library.

"Buckingham Palace, *22nd May*, 1861."

Thus encouraged, I was stimulated to greater perseverance, and opportunities were not wanting. My Work brought me numerous correspondents, many of whom are unknown to fame, and non-professional, but Mathematicians nevertheless. The majority of my correspondents have differed from me; but a minority have coincided with me, evidenced by the fact that my Oxford Pamphlet has been translated into French ; but, whether agreeing with, or differing from me, I take this opportunity of thanking all of them, for there is not one that has not rendered me good service, or perhaps I should rather say, good service to the cause of Science. When I was attending the meeting of the "British Association" at Newcastle, I received a Letter from M. Armand Grange, on the subject of the French translation of my Oxford Pamphlet, which at that period he was about publishing. I wrote him a hasty Letter in reply, giving him a diagram and what appeared to me at the moment a demonstration by means of it, which I thought might be introduced

with advantage. From a very slight omission on my part the argument is defective; both the translator and myself discovered the defect, much to our annoyance, when it was too late to rectify it; and should this Pamphlet have fallen into the hands of any of my readers, I hope they will accept this explanation, and not make use of it to raise a quibble, as some have done, aye, even after all the circumstances have been explained, and the omission given. The defect is Mathematical, and not Geometrical, and shall be rectified should we publish another edition; but I may observe, that any *Mathematician* who will be at the trouble of reading the following letter to Professor de Morgan, with the least care, will readily detect the omission, without any aid from us.

I again very freely distributed a Pamphlet at the meeting of the "British Association" at Manchester, in 1861, on which occasion G. B. Airy, Esq., the Astronomer Royal of England, presided over the Physical and Mathematical Section. The Pamphlet was addressed to the President and Members of the Committee of this Section, and I took care that a copy was in the hands of the Astronomer Royal before the business of the Section commenced. I was present when he gave his opening address, in which he made reference to the subject of my Pamphlet in terms that I very well remember. As given in the Transactions of the Section they run thus:—
"It was known to those present that great ingenuity had been employed upon certain abstract propositions of Mathematics, which had been rejected by the learned in all ages, such as finding the length of the circle, and perpetual motion. In the best Academies of Europe, it was established as a rule, that subjects of that kind should not be admitted, and it was desirable that such communi-

cations should not be made to the Section, as they were a mere loss of time." I stood up during the delivery of his address, and from my adventure in the Physical Section at the Aberdeen meeting of the Association, was very well known as the *Circle-squarer* to many present; and the raciness of the scene may, and can only be conceived, by the addition of the following, which is literally the language employed by this *great* man on that occasion. "*In electing me to the chair of this Section, you have made a despot of me for a week, and as a despot I shall act, and will take good care that no such subjects shall be brought before the Section.*" I could not help smiling, and was probably as much or even more amused, than any of those present who agreed with him in thinking me a mere geometrical myth.

I may mention one or two other incidents in connection with the Manchester Meeting of the "British Association," which may be amusing to readers, whether mathematical or non-mathematical.

A well known Philosopher was the guest of a friend of mine on that occasion, and at one of the soirees, my friend introduced me to the Philosopher, and in doing so, jocularly remarked that I was the solver of the problem of "Squaring the Circle." The Philosopher lifted up his hands and exclaimed:—Ah! My good Sir! *We* Philosophers assure you, that that is an impossibility! Very shortly afterwards another friend of mine who has a knowledge of the subject, and who has been long acquainted with the Philosopher in question, met him at a dinner-party, and happened to sit beside him. Without telling Philo that he knew me, my friend introduced the subject of the "Quadrature of the Circle," by asking him if he had read my Manchester Pamphlet, with the view of obtaining his opinion upon

that little brochure. There was some discussion between them, my friend taking the defence of my view of the question; the result of which was, that Philo admitted he had not made himself a sufficient master of Mathematical science, to be able to give an opinion upon Mr. Smith's theories.

Sir Wm. Rowan Hamilton, the Astronomer Royal of Ireland, who presided in the Physical Section at the Aberdeen Meeting of the Association, on the occasion of my reading a paper on "The Relations of a Circle inscribed in a Square," attended the Manchester Meeting. The Mathematical Section held its sittings in the "Friend's Meeting House," and on leaving the Section one day with a friend, we met Sir William on the steps of that building, and had a few minutes' jocular conversation with him. I stated that I had sent him a copy of my Pamphlet, and gave him another, and expressed a wish that he would read it carefully, and kindly write me if he could detect a fallacy. This he promised to do, and I may say that on that occasion he was the very "*personification*" of politeness, and our accidental meeting was apparently a very pleasant one. Not hearing from Sir William, and having, as I thought, some new matter worth communicating, I addressed a Letter to him, and receiving no reply, I wrote him again. The following is the copy of my second Letter:—

BARKELEY HOUSE, SEAFORTH,

NEAR LIVERPOOL, 12*th Nov.*, 1861.

DEAR SIR,—I had the honour to address a Letter to you dated the 1st inst., and as I have not heard from you in reply, I fear it may have miscarried. The following was the substance of it:—

We met accidentally in Manchester, at the meeting of the "British Association," and you will remember that we had a few minutes amusing conversation on the interesting topic of the Quadrature of the Circle. You kindly promised to give the Letter I addressed to the President and Committee of the Physical Section of the Association on that occasion a careful perusal, and I had hoped I should have heard from you, pointing out the fallacy in my facts, reasonings, or conclusions, if you had discovered that a fallacy in any of them really existed. Not having heard from you, I venture to trouble you with a few further observations on this interesting and important question.

It appears to me that the solution of the problem of the "Quadrature of the Circle" involves the consideration of three questions perfectly distinct from each other. First: Can a circle be produced of which the superficial area is exactly equivalent to the superficial area of a given square? Second: Is the ratio of diameter to circumference in a circle a commensurable relation? Third: Can the true ratio of diameter to circumference in a circle be geometrically and mathematically demonstrated?

My remarks on the present occasion shall have reference to the first of these questions only. This question I answer in the affirmative, and it appears to me that the truth of this affirmation may be demonstrated in the following way by means of any hypothetical value of $\pi$, it being admitted that $\pi$ is adopted to represent the number of times the diameter of a circle is contained in its circumference.

Let D represent the diameter, R the radius, S R the semi-radius, and A the area of circle. Then, D may be any given finite quantity, therefore, R and S R are ascertainable quantities, and both arithmetically expressible with perfect accuracy.

Now, let S R $\times$ $\pi$ represent the side of a square. Then, $(\mathrm{S\,R} \times \pi)^2$ = area of the square. Let this be a given square and let X represent it.

If D the diameter of the circle be a given finite quantity, the radius of the circle is ascertainable with perfect accuracy, and $\mathrm{R}^2 \times \pi =$ area in every circle. Now, let A be taken to represent the area of a square. Then $\sqrt{\mathrm{A}}$ must be the side of such square, and exactly equal to the diameter of an inscribed circle. $\frac{1}{2}(\sqrt{\mathrm{A}})$ = radius of such circle, and if represented by Y, then, $\mathrm{Y}^2 \times \pi$ = area of the circle, and is exactly equal to X.

For example: Let the given value of D the diameter of the circle be 8, and by hypothesis, let the value of $\pi$ be 3·1416. Then $\frac{8}{4} = 2 = \mathrm{S\,R}$, the semi-radius of the circle. $\mathrm{S\,R} \times \pi = 2 \times 3{\cdot}1416 = 6{\cdot}2832$, and $6{\cdot}2832^2 = 39{\cdot}47860224 = \mathrm{X}$. But, $\mathrm{D} = 8$, therefore, $\frac{\mathrm{D}}{2} = \frac{8}{2} = 4$ = radius of the circle; and $\mathrm{R}^2 \times \pi = 4^2 \times 3{\cdot}1416 = 16 \times 3{\cdot}1416 = 50{\cdot}2656$, = area of the circle = A. Now, let A represent the area of a square. Then; $\sqrt{\mathrm{A}} = \sqrt{50{\cdot}2656}$ must be the side of the square, and exactly equal to the diameter of an inscribed circle; $\frac{1}{2}(\sqrt{\mathrm{A}}), = \frac{1}{2}(\sqrt{50{\cdot}2656}), = \sqrt{\frac{1^2}{2^2} \times 50{\cdot}2656}, = \sqrt{\frac{1}{4} \times 50{\cdot}2656}, = \sqrt{{\cdot}25 \times 50{\cdot}2656}, = \sqrt{\frac{50{\cdot}2656}{4}}, = \sqrt{12{\cdot}5664}$, must be the radius of such circle; and $\mathrm{R}^2 \times \pi$ = area in every circle; therefore, $\sqrt{12{\cdot}5664}^{\,2} \times 3{\cdot}1416 = 12{\cdot}5664 \times 3{\cdot}1416 = 39{\cdot}47860224$, = area of the circle, and is exactly equal to X; that is, exactly equal to the area of a square of which S R $\times$ $\pi$ is the side, and any other hypothetical value of $\pi$ will produce a similar result.

I have no doubt you will give the subject matter of this Letter a careful examination, and with your usual courtesy favour me with a reply, pointing out the fallacy

in the argument, if there really be any, which I confess I am quite unable to detect.

My answers to the second and third questions are also in the affirmative, and on the receipt of your reply to this Letter, I will furnish you with demonstrations in proof of their correctness, which appear to me, and I think will also appear to you, equally simple and conclusive.

Waiting the favour of your reply, I have the honor to be, your very obedient servant,

JAMES SMITH.

SIR WM. ROWAN HAMILTON, LL.D., &c.,

Observatory, near Dublin.

It will occur to my Mathematical readers, that so far as Sir William Rowan Hamilton was concerned, I might have given him all the information contained in this Letter in much less than half the space, which is unquestionably true. For instance, having assumed D, R, SR, and A, to represent the diameter, radius, semi-radius, and area of a circle, I might have simply said :—
If $D = 8$, and $(SR \times \pi)^2 = x$, $(\frac{1}{2} \sqrt{A})^2 \times \pi = x$; therefore, $\sqrt{x} = 2\pi$, whatever the arithmetical value of $\pi$ may be; therefore, the fact is incontrovertible, that from a given square we may *mathematically* produce a circle of exactly the same superficial area. But, in the Letter I have worked out the algebraical formula by simple arithmetic, and my arithmetical readers have only to verify the calculations to convince themselves of the fact; and I cannot help thinking, that some of them will begin to suspect, that the "Quadrature of the Circle," about which there has been so much controversy in all ages, is but a school-boy question after all.

Sir William Rowan Hamilton honored me with the following reply. From the date it should have come into my hands before posting my second Letter, so that these Letters must have crossed each other, but this is a matter of little consequence.

OBSERVATORY, NEAR DUBLIN,

*November* 11, 1861.

Sir W. R. Hamilton presents his compliments to James Smith, Esquire, of Barkeley House, Seaforth, near Liverpool.

Sir W. R. H. had conceived that the correspondence between Mr. Smith and himself was closed, by his letter of the 27th April last; but he remembers perfectly the fact of his lately meeting Mr. Smith on the steps of one of the public buildings in Manchester.

He has since sought to fulfil his promise, then in politeness given, at Mr. Smith's request, that he would read part, at least, of the last paper by Mr. Smith, on the Quadrature of the Circle.

He has accordingly done so, to quite a sufficient extent, and with quite sufficient care, to be satisfied that there would be no use in his reading any further.

Sir W. R. H. regrets to say that he does not consider Mr. Smith to understand the principles of *decimal arithmetic* as applied to *infinite series*. And the fallacies, hence resulting, appear to him to vitiate the whole of Mr. Smith's arithmetical argument.

As regards the geometrical theorem, which has been known to Mathematicians for about two thousand years, that *eight circumferences of a circle exceed twenty-five diameters*, it has been recently confirmed by Sir W. R. H. in an elementary demonstration, which he may perhaps be induced to republish.

But as it must be obviously useless to continue, or rather to re-open this correspondence, Sir W. R. H. hopes that Mr. J. Smith will not consider him as discourteous, if he shall not in future acknowledge any printed or other communication on the subject.

JAMES SMITH, ESQ.,
Barkeley House, Seaforth, near Liverpool.

This is obviously no answer to my Letter, and was of course not intended to be. It is carefully written, and so indited as to prevent the possibility of re-opening our

correspondence. I thus found that there was no hope of aid from this quarter, in the interesting enquiry in which I was engaged, and our acquaintance terminated with my reply, which ran as follows:—

BARKELEY HOUSE, SEAFORTH,

NEAR LIVERPOOL, *15th November*, 1861.

Mr. James Smith presents his compliments to Sir William Rowan Hamilton, LL.D., &c., &c., of the Observatory, near Dublin, and "The Astronomer Royal of Ireland," and begs to apologize for the delay in acknowledging the receipt of Sir W. R. H.'s note, for which he could give a good reason.

Mr. Smith observes the distinction drawn by Sir W. R. H. between meeting Mr. S. *at the British Association* in Manchester, and meeting him *on the steps* of one of the public buildings of Manchester, the public building in question being the one in which the Physical Section of the Association held its meetings on that occasion; and Mr. Smith regrets to find that Sir W. R. H., who, he had been led to believe was the most polite and polished gentleman in Ireland, can, when it is convenient to do so, play a part greatly at variance with his general character.

Mr. Smith could prove, if necessary, through the gentleman to whom Sir W. R. H. appealed, to evidence the fact of his disbelief in Mr. Smith's theory, that Sir W. R. H. did promise to give the Letter Mr. S. had addressed to the President and Committee of the Physical Section of the British Association a careful perusal; and Mr. S. regrets to find that the politeness of Sir W. R. H. on that occasion, would appear to have been a mere cloak to conceal his want of candour.

Mr. Smith observes that Sir W. R. H. considers *a knowledge of decimal arithmetic as applied to infinite series* "*essential*," for the purpose of demonstrating the "geometrical theorem"* of how many

* The reader will observe, that Sir William Rowan Hamilton considers, that the "geometrical theorem" of how many times the diameter of a circle is contained in its circumference, can only be solved by one who has a knowledge of "*decimal arithmetic as applied to infinite series.*" To establish this assumption, Sir William must prove the following algebraical formula to be false, which he will find to be an impossibility. If the diameter of a circle be 12, and $3\,(SR \times \pi) = X$, $\frac{R^2 \times \pi}{4} = X$; therefore, X = the difference

times the diameter of a circle is contained in its circumference; and Mr. S. cannot but express his surprise that Sir W. R. H. should be so inconsistent, as to profess to recognise, and believe in, the "authority" of the Mathematicians of two thousand years ago, who certainly had no knowledge whatever of "*decimal arithmetic as applied to infinite series.*"

Mr. Smith would not have again intruded himself upon Sir W. R. H., if he had not had new matter to communicate, which he thought was well deserving of consideration by the highest "authorities" in Mathematical Philosophy. To Mr. S. it is a matter of indifference that Sir W. R. H. declines to re-open a correspondence with him, and would only remark that Sir W. R. H. may yet live to acknowledge the fact, that "the wisdom of the wise may be destroyed, and the understanding of the prudent brought to nothing."

Mr. Smith has only to remark in conclusion, that he will spare Sir W. R. H. the pain of being either discourteous or uncandid for the future, so far as he is concerned. It is quite sufficient for Mr. S., for the convenience of regulating his future course of procedure, to have discovered, that Sir W. R. H. can be both discourteous and uncandid, when it suits his purpose to display these qualities.

SIR W. ROWAN HAMILTON, LL.D., &c.,

Observatory, near Dublin.

I once more freely distributed a Pamphlet at the meeting of the "British Association" at Cambridge in 1862. This Pamphlet was addressed to the President and Vice-presidents of the Association, which included in its list some of the greatest "Mathematical Authorities" of the day: such as Dr. Whewell, the Master of Trinity; the Astronomer Royal; Professor Adams, the competitor of Le Verier for the honour of the discovery of

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between the area of a circle of which the diameter is 8, and the area of a circle of which the diameter is 10; whatever the arithmetical value of $\pi$ may be. How truly has Professor de Morgan told us, that "*crammed erudition does not cast out any hooks for more.*" Poor Sir William! How unfortunate! What a pity he should have to make up his mind, like my correspondent, Lieut.-General T. Perronet Thompson, to be looked upon in the future by every first class school-boy, as a mere simpleton, notwithstanding the profundity of his "*mathematical wisdom.*"

the last new planet; and others of scarcely less eminence. I had prepared this Pamphlet some time before the meeting of the Association, and sent copies to those to whom it was especially addressed. The Dean of Ely, one of the list, could tell an amusing story as to the history of this Pamphlet, and he has my permission to do so, if he thinks it worth while. Dr. Whewell acknowledged the receipt of the Pamphlet in an exceedingly polite note, so polite indeed that I was tempted to write him a Letter in reply, of such a length that were I to give a copy of it I should be carried beyond the limits I have allotted to myself, and I may assure the reader that it is not absolutely necessary. To this Letter I received the following reply :—

THE LODGE, CAMBRIDGE,
*September 14th*, 1862.

SIR,—I have received your explanation of your proposition that the circumference of the circle is to its diameter as 25 to 8. I am afraid I shall disappoint you by saying that I see no force in your proof: and I should hope that you will see that there is no force in it if you consider this :—In the whole course of the proof, though the word circle occurs, there is no property of the circle employed. You may do this: you may put the word *hexagon* or *dodecagon*, or any other word describing a polygon in the place of *Circle* in your proof, and the proof would be just as good as before. Does not this satisfy you that you cannot have proved a property of that special figure—a circle?

Or you may do this: calculate the side of a polygon of 24 sides inscribed in a circle. I think you are a Mathematician enough to do this. You will find that if the radius of the circle be one, the side of this polygon is ·264 &c. Now, the arc which this side subtends is according to your proposition $\frac{3 \cdot 125}{12} = \cdot 2604$, and therefore the chord is greater than its arc, which you will allow is impossible.

I shall be glad if these arguments satisfy you, and

I am, Sir, your obedient Servant,

W. WHEWELL.

The argument in the first paragraph of this Letter will appear plausible to the uninitiated; but it is literally absurd, unless, indeed, the Doctor is prepared to prove that a *hexagon* or *dodecagon, or any other figure describing a polygon,* can be made to enclose as large a surface as a circle of equal perimeter. The same fallacy lies at the root of the Doctor's second argument; but I am willing to admit that the absurdity of it is not equally apparent, and requires thought and reflection to detect it. I cannot give my reply to the Doctor's Letter, *in extenso*, but I make the following quotation from it:—"It cannot be disputed that if 8 circumferences of a circle are exactly equal to 25 diameters, which makes $\frac{25}{8} = 3{\cdot}125$ the arithmetical value of $\pi$, the area of a circle of which the diameter is unity, is $\frac{3{\cdot}125}{4} = {\cdot}78125$. Now, if the area of a circle of which the diameter is one, be greater than ·78125, say ·7854 (its approximate value on the Orthodox theory), it is obvious, that whether the circumference or diameter of a circle be the given quantity to find the area, the area of the circle should be greater if calculated on the latter, than if calculated on the former theory; but this is not a fact. I have proved in my printed Letter, by taking a polygon of 25 sides inscribed in a circle, that every circle contains a larger area (on the theory that 8 circumferences are equal to 25 diameters), than it can be made to contain on the theory that $\pi = 3{\cdot}1416$, which makes the area of the circle of which the diameter is unity ·7854. If you will do me the honour to read my printed Letter from page 42 to near the end, bearing in mind the facts with which you are perfectly familiar, that the circumference of a circle is to the perimeter of an inscribed regular hexagon, as the area of the circle to the area of an inscribed regular dodecagon; I cannot help thinking that '*you will be satisfied*' of the impossibility of ascertaining the ratio of diameter

to circumference, or the ratio of diameter to area in a circle, even approximately, by means of a succession of inscribed polygons. For, the polygons are obtained by the addition of promiscuous right angled triangles to every successive polygon, and these triangles have no exact relation to the circle or to each other, either in perimeter or area."

Dr. Whewell took no notice of this Letter, and on the 12th Oct., I wrote him again. It would somewhat detract from the prominence I wish to give to my Letter to Professor de Morgan, if I were to give this communication at length, and it will be obvious to the reader, from the following reply, that it is quite unnecessary.

TRINITY LODGE, CAMBRIDGE,
OCTOBER 16, 1862.

SIR,—I did not answer your second letter, because I thought that in my reply to your first letter I had said all that it was necessary for me to say on the subject on which you wrote. I am asked opinion still: and in reply to your third letter I must refer you to the letter I have already sent you. My letter was written as a private one in the hope of setting you right. Though I have failed in that object, you are at liberty to publish it if you please; and this also. I am, Sir, Your obedient Servant,

JAMES SMITH, ESQ. W. WHEWELL.

So the *great* Dr. Whewell made his stand upon the 24 sided polygon inscribed in a circle of diameter unity, vainly imagining that in his first Letter he had said enough to prove, that the theory which makes 8 circumferences equal to 25 diameters in every circle, makes a chord greater than its subtending arc, which any *Mathematician* will admit to be an absurdity. This argument is the only one that Orthodoxy has to offer in its defence, and I may tell the reader that the majority of my correspondents have made the 24 sided polygon the basis of their reasoning. It is true, that some of them

have adopted the 64 sided, and one, and one only, the 60 sided polygon. I have answered this objection over and over again, and it is a *remarkable fact*, that not one of my corre spondents have ever attempted to grapple with my arguments; on the contrary, they have all *invariably* treated them with contempt; one of them fancying forsooth, that he had vitiated them by boldly daring to assert, that I am a "*mere constructor of traps to catch myself.*" Well! well! I fearlessly tell Dr. Whewell, and every other hero of the multilateral sided inscribed polygons to a circle, that if they will read my Letter to Professor de Morgan, they will find an unanswerable exposure of the fallacy that lies at the root of their argument.

I took a deep interest in the proceedings of the Association at Cambridge, and was present when the President, Professor Stokes, gave his opening address to the Physical Section. The Rt. Hon. Lord Wrottesly, the Astronomer, and President of the Association in 1860, was sitting beside him. I have not the slightest objection to offer against the tone of the Professor's remarks, as to how subjects of a "*very abstract character*" should be treated by that Section; but the effect of his arguments was precisely the same as if he had chosen to adopt the style of the Astronomer Royal of England. From his observations I was induced to address a Letter to Lord Wrottesly, of which the following was the opening paragraph. "As a very old Life Member of the British Association, I venture to address a few lines to your Lordship on an abstract mathematical question of interest and importance, and hope that your Lordship will not deem me intrusive in taking this liberty." The concluding paragraph ran as follows:—"I was in Section *A* this morning, and heard the opening address of the President, and I noticed that your Lordship was also

present and an attentive listener to his observations. I have hitherto thought it strange that a rule of the Association should prohibit the introduction of this subject, (The Quadrature of the Circle), for consideration in the Mathematical and Physical Section; but probably the President is right, and this is one of those abstract mathematical questions which is better suited to quiet discussion, and '*the mutual interchange of ideas between two different minds;*' and it was the hint thus received, that has induced me to address your Lordship, as one of the best non-professional mathematical authorities of the Association, on this important subject; and I hope your Lordship will do me the honor to reply to this communication." I added the following as a postscript:—"I addressed a Letter to the President of Section *A* yesterday, giving him an entirely different mode of demonstration. I should be glad if you would compare notes with the Professor, or, if you prefer it, I will furnish you with a copy of the Letter.

Remembering that the late and ever-to-be-lamented Prince Consort had told us, in his opening address at Aberdeen, that the British Association was "*not a secret confraternity of men, jealously guarding the mysteries of their profession; but a popular Association, inviting the uninitiated, the public at large, to join them, having as one of its objects to break down those imaginary and hurtful barriers, which exist between men of science and so-called men of practice.*" And I, innocently supposing that the Members of the Association met for the "*mutual interchange of ideas*" on subjects connected with Science, and met as I imagined for the time being, on a footing of equality, ventured to speak to his Lordship at one of the soirees, and referred to the Letter I had addressed to him. But, on that occasion his Lordship very speedily

gave me to understand, that his opinions were anything but in harmony with such like *new-fangled* notions. I never heard from his Lordship, or Professor Stokes, and received no answers to other Letters, addressed during the meeting, to several of our great Mathematicians. I mention these incidents to you, Reader, to show how little I have been aided in my enquiry, by our "*known and recognised authorities*" in Mathematical and Geometrical Science; whether professional or non-professional; and to show further, how carefully the "*Mathematical confraternity*" have laboured, to make a suggestion "*barren and unfruitful in the mind of the original suggester.*" (*See Prof. Stoke's address, in the Transactions of the Association.*)

Since the meeting at Cambridge, I have given the British Association very little trouble, and the only incident worth mentioning in connexion with the Association is the following:—At the last meeting at Bath, I distributed a Letter, accidentally drawn from me only a few days before. M. J. Whitty, Esq., the Proprietor of the Liverpool "Daily Post," had frequently twitted me on the subject of "Squaring the Circle," with the question:—*Cui bono?* and had more than once said to me—if you can, why do you not show a practical application of it? He repeated this on a particular occasion in the presence of certain gentlemen, who were of opinion that I not only could, but had given more than one practical application of it, and who told him so. This led to a conversation, and ultimately to a request on his part, that I would give him the substance of that conversation in the shape of a Letter, and he would see if attention could not be drawn to the subject. Mr. Whitty found himself as powerless as the writer to entice the "*authorities*" out of their shell, and beyond appearing in the "Daily Post," the Letter has remained unnoticed to this hour. (*See Appendix C.*)

Reader, if you happen to have seen my last Pamphlet, entitled, "A Nut to Crack," you will be aware that it is quite unconnected with the (so-called) "*British Association for the Advancement of Science.*" You will have observed that it originated from the "Budget of Paradoxes," with which Professor de Morgan is favoring the Scientific world through the columns of the "Athenæum." The Professor is an old bird and not to be easily caught, and by no efforts of mine have I been able, up to the present moment, either to induce or twit him into a discussion of the question at issue between me and the mathematical world; but after all, this is a matter of but little consequence, as time will prove. The readers of that Pamphlet will also be aware, that Lieutenant General T. Perronet Thompson made an attack upon me through the Press, in which he made a display of his profound ignorance that such a thing existed, as a theory with reference to commensurable right angled triangles, and which led to a long correspondence between us. The publication of the Pamphlet in question suspended our correspondence for a time, but it was subsequently renewed. Professor de Morgan fixes the General's position in the ranks of paradoxers, and consequently, I presume he is not to be recognised as a "*Mathematical authority.*" Be this as it may, I can say this for the General, he is one of the best "*literary fencers*" that ever took pen in hand, not excepting the Professor himself; not one of all my numerous correspondents are to be compared with him in this respect, although many of them have displayed considerable skill in the art of *literary fencing.*

Well, then, the General, like the Master of Trinity, entrenched himself behind the 24 sided polygon inscribed in a circle, and I certainly found it difficult, from his

clever generalship, to get at him. His steady determination to admit nothing, and his firm resolution to evade a distinct answer to a plain question, for a long time baffled me. We did, however, at length get to close quarters, and between the 25th July and the 3rd September, 1864, we exchanged no less than seventeen Letters, not a day passing (Sundays excepted) without one or other of us firing a shot. The shot I let off on the 30th August, 1864, took the following direction.

BARKELEY HOUSE, SEAFORTH,
LIVERPOOL, 30*th August*, 1864.

DEAR SIR,—"You may depend on it," there is much in the argument advanced in my last Letter, notwithstanding your inability to comprehend it; and I deny your assertion that it "disproves itself."

Now, my good Sir, you may take any number and its half, and without the aid of geometry, you may multiply them in and in, and by extracting roots make them meet in an arithmetical quantity with non-terminating decimals, which may be extended as far as you please; and this is just what happens when you calculate the area of a circle from four sided circumscribed and inscribed polygons; and you may rely on it, that it is sheer nonsense to suppose the extremes meet in the area of an inscribed circle to a *square*, the area of such square being represented arithmetically by the larger number.

Permit me to show you how to make a right use of inscribed polygons to a circle, as a means of ascertaining the true ratio of diameter to circumference in a circle.

Let A, B, C, D, and so on to the end of the alphabet, represent a series of inscribed circles to a series of diminishing squares inscribed within each other, and let the diameters of the circles be to each other in the proportion of 25 to 24. That is, let the diameter of the circle A be to the diameter of the circle B, as the

diameter of the circle B to that of the circle C, and the diameter of the circle C to that of the circle D. Then, as respects their diameters, A is to B, B to C, and C to D, as 25 to 24; and so on we might proceed, and the diameter of each circle would be to the diameter of its adjacent smaller circle, in the ratio of 25 to 24; and to the diameter of its adjacent larger circle in the ratio of 24 to 25; and by adopting new symbols when we arrive at the end of the alphabet, it is perfectly obvious, we might go on, *ad infinitum.*

Then, the circumference of the circle B will be equal to the perimeter of a regular six sided inscribed polygon to the circle A; the circumference of the circle C will be equal to the perimeter of a regular six sided inscribed polygon to the circle B; and the circumference of the circle D will be equal to the perimeter of a regular six sided inscribed polygon to the circle C, and so on, *ad infinitum.* As a Geometer and Mathematician you can have no difficulty in convincing yourself of these facts. You have the explanation of them in the short paragraph at the end of my Letter of the 27th instant, in which they are Geometrically and Mathematically traced to their cause or root.*

Well then, let the diameter of the circle A be any

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* The following is the paragraph referred to:—"Let me induce you to master the argument in my two last letters; and pray reflect upon the consequences flowing from the following incontrovertible facts:—If $x = 3$, and $y = \frac{3}{4\pi} = \frac{3}{12{\cdot}5} = {\cdot}24$; it follows of necessity, that $\frac{x \div y}{4} = \frac{25}{8}$; that is, $\frac{3 \div 24}{4}$ and $\frac{25}{8}$ are equal, and both equal to $3{\cdot}125$, $= \pi$, $=$ circumference of a circle of which the diameter is unity; therefore, $\frac{x \times y}{4y} = \frac{3 \times {\cdot}24}{4 \times {\cdot}24} = \frac{{\cdot}72}{{\cdot}96} = {\cdot}75 =$ area of a regular dodecagon or twelve sided regular polygon inscribed in a circle of which the diameter is unity. And, $\frac{\text{area of the dodecagon}}{4y} = \frac{{\cdot}75}{{\cdot}96} = {\cdot}78125 = \frac{\pi}{4} = \frac{3{\cdot}125}{4} =$ area of a circle of which the diameter is unity."

given quantity, say 60. Then $6\left(\frac{60}{2}\right) = 6 \times 30 = 180$, = the perimeter of a regular six sided inscribed polygon to the circle A ; $\frac{24}{25}(60), = \frac{24 \times 60}{25} = 57{\cdot}6$, = diameter of the circle B ; and $6\left(\frac{57{\cdot}6}{2}\right), = 6 \times 28{\cdot}8, = 172{\cdot}8$, = the perimeter of a regular six sided inscribed polygon to the circle B; and, $172{\cdot}8 : 180 :: 3 : 3{\cdot}125$. Now, the circumferences of circles are to each other as their diameters; and the perimeters of regular polygons are to each other as the circumferences of their circumscribing circles ; and the fact is incontrovertible, that the perimeter of a six sided regular polygon is to the circumference of its circumscribing circle, in the ratio of 3 to $\pi$, whatever the arithmetical value of $\pi$ may be.

Now, by hypothesis, let $\pi = 3{\cdot}1416$. Then, $60 \times 3{\cdot}1416 = 188{\cdot}496$ = circumference of the circle A ; and $57{\cdot}6 \times 3{\cdot}1416 = 180{\cdot}95616$, = circumference of the circle B ; and $180{\cdot}95616$ is to $188{\cdot}496$, *not* in the ratio of 3 to the Orthodox value of $\pi$, but in the ratio of 3 to a value of $\pi$, which makes 8 circumferences of a circle exactly equal to 25 diameters, and makes $\frac{25}{8} = 3{\cdot}125$ the value $\pi$; for, $180{\cdot}95616 : 188{\cdot}496 :: 3 : 3{\cdot}125$ ; and proves the truth of my theory. And pray let these facts be taken in connection with the arguments and proofs advanced in my two or three last Letters.*

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* Hence, if the circumferences of two circles are in the proportion of $3{\cdot}125$ to 3, or getting rid of decimals, in the proportion of 25 to 24; the circumference of the smaller circle is equal to the perimeter of a regular inscribed hexagon to the larger circle.

I might and ought to have strengthened this argument in the following way:—The diameter of the circle A is, by hypothesis, = 60, therefore, $\frac{60}{2} = 30$, = radius, and $r^2 \times \pi$ = area in every circle, therefore, $30^2 \times \pi = 900 \times 3{\cdot}125 = 2812{\cdot}5$ = area of the circle A. Now, the area of a regular inscribed dodecagon or twelve sided regular polygon to the circle A = 6 (radius × semi-radius), therefore, $= 6\left(30 \times \frac{30}{2}\right) = 6\,(30 \times 15) = 6 \times 450 = 2700$

It is truly hard, as you say, to "*kick against the pricks*," and so you will find it, when, instead of meeting argument with dogmatical assertion, and sophistical evasion, you grapple with it; and for your own sake I shall regret exceedingly, if you any longer persevere in your determination to act on so unwise and absurd a principle, as that you have hitherto adopted.

I remain, dear Sir,

Yours very sincerely,

LIEUT.-GEN. T. PERRONET THOMPSON, BLACKHEATH. JAMES SMITH.

I found, during the course of our correspondence, that the General, like all our great Mathematical authorities, could work up his imagination into the belief of the most extraordinary fancies; and on this occasion he had worked it up into the belief of the *exceedingly great fancy*, that the following communication was an excellent return shot.

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= area of a regular dodecagon inscribed in the circle A. But, the diameter of the circle B = 57·6, therefore, $\frac{57 \cdot 6}{2} = 28 \cdot 8$ = radius. Or, the circumference of the circle B = 180, and $\frac{\text{circumference}}{2\,\pi}$ = radius in every circle, therefore, $\frac{180}{2\,\pi} = \frac{180}{6 \cdot 25} = 28 \cdot 8$ = radius of the circle B; and $r^2 \times \pi$ = area in every circle; therefore, $28 \cdot 8^2 \times \pi = 829 \cdot 44 \times 3 \cdot 125$; = 2592 = area of the circle B; and 2592 : 2700 : : 2700 : 2812·5; therefore, the area of the circle B is to the area of an inscribed regular dodecagon to the circle A, as the area of the dodecagon to the area of the circle A. This is a beautiful illustration of the meeting of Geometrical and Mathematical extremes. Dr. Whewell can make the calculations on some other value of $\pi$, than that which makes 8 circumferences of a circle exactly equal to 25 diameters; the Orthodox value of $\pi$ for example; and I ask him to mark, and reflect upon, the absurdity to which it leads. The Doctor may not wish to be convinced, as it must necessarily "*disturb existing systems*," but I defy him to controvert the argument, or *disturb* the conclusion. This argument has never been answered by any of my correspondents.

ELIOT VALE, BLACKHEATH,
LONDON, 31*st August*, 1864.

DEAR SIR,—It is question of admitting that twice 9 is 18, but twice 10 is less. So there is no more to be said.

Yours sincerely,
T. PERRONET THOMPSON.

JAMES SMITH, ESQ.,
BARKELEY HOUSE, SEAFORTH.

It will require no observations of mine to convince the Reader, that it was impossible to carry the correspondence farther; and I acknowledged this communication in the following terms, which I imagined would have been my last Letter to the General on this subject:—

BARKELEY HOUSE, SEAFORTH,
LIVERPOOL, 1*st September*, 1864.

DEAR SIR,—In taking my final leave of you I may observe, that I cannot help regretting, for your own sake, that you should have resolved to pertinaciously persist in meeting argument with dogmatical assertion, predetermined evasion, sophistical equivocation, and a supercilious assumption of Mathematical infallibility; and in contemptuously asserting that the question at issue between us is one of admitting that twice 9 is 18, but twice 10 is less, you furnish a splendid key-stone to the arch of Mathematical folly.

Wishing you health and happiness for many years to come, and with thanks for the services you have rendered me in the interesting enquiry in which we have been engaged.

I remain, dear Sir,
Yours very sincerely,
JAMES SMITH.

LIEUT.-GEN. T. PERRONET THOMPSON,
BLACKHEATH.

You may readily conceive, Reader, how much I was amused at receiving the following reply to this Letter; and I am disposed to think you will agree with me in considering it a rich gem, and that it will help to perpetuate the General's memory:—

ELIOT VALE, BLACKHEATH,
LONDON, *2nd September*, 1864.

Dear Sir,—I cannot omit thanking you for a great deal of useful gymnastic, which is the soil out of which all conviction must grow.

Yours very sincerely,
T. PERRONET THOMPSON.

JAMES SMITH, Esq.,
BARKELEY HOUSE, SEAFORTH.

My correspondence with Lieut.-Gen. T. Perronet Thompson, terminated with my reply to this communication, which ran as follows:—

BARKELEY HOUSE, SEAFORTH,
LIVERPOOL, *3rd September*, 1864.

Dear Sir,—Understanding the term "*gymnastic,*" as you have applied it in your favor of yesterday, to mean the exercise of intellect, "I cannot omit thanking you" for the very high compliment you have paid me.

I remain, Dear Sir,
Yours very sincerely,
JAMES SMITH.

LIEUT.-GEN. T. PERRONET THOMPSON,
BLACKHEATH.

The following is a quotation from one of my Letters to Professor de Morgan. "Well then, my good Sir, I agree with you that "*crammed erudition does not cast out any hooks for more*"; indeed, so far as my experience

goes, it would appear as if '*crammed erudition*' in Mathematical Science, carried humanity into a region where man ceases to be a reasoning being, and becomes transformed into an animal with a head so '*crammed*' with uncommon sense, that there is not a cranny left in it for common sense to find a resting place." So far, subsequent experience does not enable me to retract the opinion; and it remains to be seen, whether those who assume to be our "*guides and authorities*" in Mathematical and Geometrical Science, will afford me the opportunity; of which I shall not only be ready and willing, but delighted to avail myself. In my last Pamphlet, I gave them a Mathematico-geometrical Nut to Crack, and so far, they have not attempted to get at the kernel. Their silence leads to the inference, that they find themselves incompetent to the task; and I have therefore taken upon myself to crack it for them, in the following Letter to Professor de Morgan.

In conclusion I may observe, that at the last meeting of the British Association, I was one of a large party of its Members on a visit to the extraordinary Druidical remains at Stanton Drew, about 11 miles from Bath. I subsequently addressed a letter to the Editor of the "Times," which he declined to insert. This Letter I have given in the appendix D, which can hardly fail to be of immediate interest to the Antiquary; and may at some future day raise an interesting question among Geometers and Mathematicians, whether after all, I am the real discoverer of the true ratio of diameter to circumference in a circle.

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
Near LIVERPOOL, 1*st May*, 1865.

THE

# QUADRATURE OF THE CIRCLE.

# LETTER

TO

## PROFESSOR DE MORGAN,

FOR THE

ESPECIAL EDIFICATION OF THE READERS OF HIS

"BUDGET OF PARADOXES."

SIR,—On the 10th of October, 1863, there appeared in the columns of the "Athenæum" the introductory chapter to a series of articles, which have since appeared from time to time, latterly at rather long intervals, and apparently without any sufficient reason for it. These articles are from your pen, are entitled "A Budget of Paradoxes," and you tell us your "intention in publishing this budget, is *to enable those who have been puzzled by one or two discoverers, to see how they look in the lump.*" Pray, Sir, what is the *real* meaning of your intention? Do you employ the word discoverers ironically? If not, may I ask:—Of what are they discoverers? I have been a careful reader of your budget, and it certainly appears to me, your intention would have been better expressed if you had said, your object was to prove all

circle-squarers, with certain other fry, to be fools, and no discoverers at all. Be this as it may, I have addressed you from time to time, as your budget has afforded me the opportunity; but, I am a circle-squarer, and you had said you would never again "*talk to any squarer of the circle,*" and I presume you thought you must be as good as your word; consequently, you have never acknowledged the receipt of any of my communications, and I might naturally suppose they had been laid aside, "*to be read when you please,*" as you tell us is your wont. I cannot help thinking, however, that your friend Lieutenant General T. Perronet Thompson has been a little disappointed at your pertinacious silence, for he and I were in correspondence, and our correspondence was suspended for a time, that I might have "*a passage at arms*" with you; and the General told me he should look forward to it with great interest. *(For my last Letter to the Professor, see Appendix A.)*

In your "Budget" of the 19th March, 1864, you make Lauder who disputed some of Newton's theories into a paradoxer, and in your very short comment upon his work, you compare Newton with Goliah, and Lauder with David. You say David had five pebbles, and Lauder five arguments, and you give the Apostle Paul a place in the world's history, for which *he* would not thank you. This particular Budget induced me to address a Letter to you, dated 31st March, 1864, from which I make the two following quotations. First: "You have not done me the honour to reply to my Letter of the 5th instant, but after all, you do read some of my communications, of which there is distinct evidence in your Budget of the 19th instant, in which you make an attempted hit or two at *particular* discoverers, which will hardly be observable to the generality of your

readers. For instance, which of your readers would know who you had in your mind's eye when you penned the passage in which you say :—'Accordingly, we have plenty of discoverers, who, even in Astronomy, pronounce the learned in error because of Mathematics'; but you and I know to whom you refer in particular." (*See Appendix B.*) Second : "Now, my good Sir, although in your own and the world's estimation you are a *Mathematical Goliath,* it would appear you have resolved to play the part of a Mathematical snail, and keep within your shell; but I venture to tell you that you will not escape the fate of Goliath : the pebbles from the sling of simple truth and common sense, will ultimately crack your shell, and put you '*hors de combat*' in the fight on Circle-squaring."

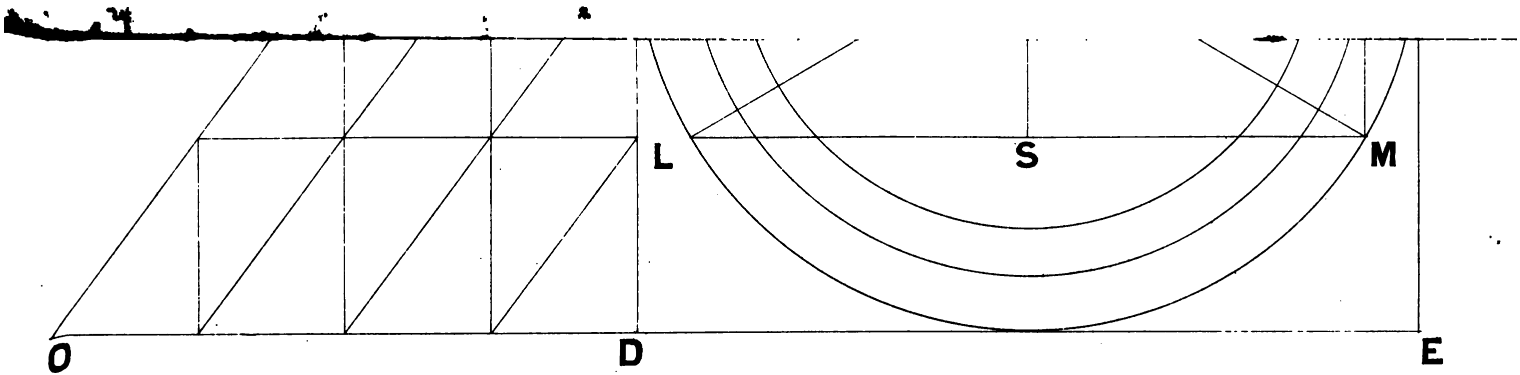
And now, Sir, I frankly tell you, that since I penned the Letter from which these quotations are extracted, many Mathematical pebbles have been thrown into my casket; and I am thankful that Providence has gifted me with the sling of common sense. My duty is to make a proper use of these gifts in the interest of mankind, and I shall now proceed to employ them in such a way, as will leave you and other *Mathematical Goliaths,* prostrate on the battle field of Circle-squaring, and lay the foundation of many important discoveries in Astronomical, Nautical, and Mechanical Science.

Without further preface, I beg to direct your especial attention to the annexed diagram, of which the following is the construction :—

With A as centre and any interval, describe the circle X; and with B as centre and the same interval, describe the circle Y. Join B A, and about the circle X circumscribe the square C D E F, making C D a side of the square parallel to B A. From the point K where the circumferences of the circles cut each

other, draw a straight line through A the centre of the circle X to meet the circumference at the point L. From the point K let fall a perpendicular to meet the circumference of the circle X at the point M. Join M L, A M, and A F. From the point A draw a straight line parallel to L M, to meet and bisect the line K M at the point N. Produce B A to meet and bisect the line L M at the point S. Produce the line E D to O, making D O equal to B S and join C O, producing the right angled triangle C D O. From the point K draw a straight line parallel to C O, to meet the line A N at the point P, producing the right angled triangle K N P. With A as centre and $\frac{1}{2}$ (B S) as interval, describe the circle Z; and with A as centre and A N as interval, describe the circle X Y Z.

Now, the triangle K M L is a right angled triangle, of which K M the perpendicular is equal to the half of K L the hypothenuse, and this triangle is the root and foundation of Trigonometrical Science. $BA + AS = BS$, and $A\,S = \frac{1}{2}\,(B\,A)$; therefore, A S = semi-radius of the circle X; therefore, the line B S is equal to three fourth parts of the line C D, a side of the circumscribing square to the circle X, by the construction of the diagram, of which any *Geometrician* may readily convince himself. If any man making pretensions to be a *Geometer* should have the temerity to dispute these facts, he would simply write himself down a fool; and if any recognised "*Mathematical authority*" should have the audacity to raise a quibble with regard to them, he would proclaim himself a knave. Well then, because D O and B S are equal, by construction, $D\,O = \frac{3}{4}\,(C\,D)$; and because K P is parallel to C O, by construction, the right angled triangles C D O and K N P are similar triangles, and the sides containing the right angle, are in the proportion of 3 to 4.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Now, Sir, I beg to observe, that whatever you may be pleased to say or think of me, I most distinctly repudiate your absurd assertion, that I am competent "*to reason upon no premisses at all;*" and I shall therefore proceed to lay down certain premisses with regard to the Geometrical figure represented by the diagram, reason them out to their legitimate conclusions, and throw out the challenge to you and every other "*Mathematical Goliath,*" to controvert the arguments by which I arrive at them.

First premiss: The angles N A G and G A B are angles of $45^\circ$, and $\frac{\pi \times 45^\circ}{180^\circ} = \frac{\pi}{4}$, whatever the arithmetical value of $\pi$ may be; and it follows of necessity, that $\frac{\pi}{4}$ is an expression of the circular measure of an angle of $45^\circ$ to a circle of which the radius is 1, and is represented by symbols which also express the value of the area of a circle of which the diameter is unity. For example: If $\pi = 3{\cdot}1416$, then, $\frac{3{\cdot}1416 \times 45}{180} = \frac{141{\cdot}372}{180} = \frac{\pi}{4} = \frac{3{\cdot}1416}{4} = {\cdot}7854$; = the circular measure of an angle of $45^\circ$ to a circle of radius 1; and is also equal to the area of a circle of which the diameter is unity, on the hypothesis that $\pi = 3{\cdot}1416$; therefore, on this hypothesis, 8 times the circular measure of an angle of $45^\circ$ to a circle of radius 1, $= 8 \times {\cdot}7854 = 6{\cdot}2832 = 2\pi =$ circumference of a circle of radius one; and any other hypothetical value of $\pi$ intermediate between 3 and 4, will produce a similar result. Now, the *Natural sine* of an angle of $45^\circ$ is equal to half the side of an inscribed square to a circle of radius 1, $= \frac{1}{2}(\sqrt{2}) = \sqrt{\frac{2}{4}} = \sqrt{{\cdot}5}, = {\cdot}707107$, and is so given in the Mathematical Tables of *Natural sines;* and so far there can be nothing to quarrel about, between me and any Mathematician.

Second premiss: The angle N A K is an angle of $30^\circ$, and KN the *sine* of the angle is equal to half the side of an inscribed regular hexagon to a circle of radius 1, $= \frac{1}{2} = \cdot 500000$. This is the *Natural sine* of the angle N A K, and is so given in the Mathematical Tables of *Natural sines.* But, 6 times the *Natural sine* of the angle N A K $= 6 \times \cdot 5 = 3 =$ perimeter of a regular inscribed hexagon to a circle of which the diameter is unity; and the circumference of a circle of which the diameter is unity is represented by $\pi$; therefore, since the property of one circle is the property of all circles, it follows of necessity, that $\frac{\pi}{3}$ expresses the ratio between the circumference of every circle, and the perimeter of its inscribed regular hexagon, whatever the arithmetical value of $\pi$ may be. There is more in this very plain and simple fact, than was ever dreamed of in the Philosophy of our "*Mathematical Goliaths,*" as shall be made manifest before I conclude this Letter.

Third premiss: For practical purposes we assume the circumference of a circle to be represented by $360^\circ$; and the Algebraical formula $(\frac{\pi}{4} \times 100) : \pi :: 360^\circ : \frac{360^\circ}{25}$ is infallible; whatever Arithmetical value we may be pleased to put upon $\pi$, intermediate between 3 and 4. For example: If we assume the value of $\pi$ to be $3 \cdot 1416$, then, $(\frac{3 \cdot 1416}{4} \times 100) : 3 \cdot 1416 :: 360^\circ : 14^\circ\ 24'$, and any other hypothetical value of $\pi$ will produce a similar result; therefore, it follows of necessity, that 100 times the circular measure of an angle of $45^\circ$ to a circle of radius one, is to $\pi$, as $360^\circ$ to $\frac{360^\circ}{25}$, whatever the true arithmetical value of $\pi$ may be.

Now, Sir, I tell you without any hesitation, that from these premisses, which are incontrovertible, and which no *Geometrican* and *Mathematician* would dare to dispute; we may establish beyond the possibility of dispute or cavil,

that 8 circumferences are exactly equal to 25 diameters in every circle, which makes $\frac{25}{8} = 3 \cdot 125$ the true arithmetical value of $\pi$. This I shall now proceed to prove.

You yourself admit, Sir, that the "*logic of Mathematics is certainly that of common life*," although as you say, and say truly, "*the data are of a different species and admit of no doubt.*" Well then, it follows of necessity, from your own admissions, that if our premisses be sound, and our reasoning logical, our conclusion must be true; and it will be for you to detect a fallacy in the proofs I am about to introduce to your notice; and if you find yourself incompetent to the task, which most assuredly you will do; then I say to you:—Take courage! Creep out of your shell and shew yourself! Have the manliness to speak out and admit, that the solution of the problem of "Squaring the Circle" is "*un fait accompli!*"

In the first place: It cannot be disputed, that 8 times the circular measure of an angle of $45°$ to a circle of radius 1, is equal to the circumference of the circle $= 2\pi$, whatever the arithmetical value of $\pi$ may be. But, $\frac{100}{8} = \frac{50}{4} = 12 \cdot 5$; and $12 \cdot 5 \times \frac{\pi}{50} =$ the circular measure of an angle of $45°$ to a circle of radius 1, whatever the arithmetical value of $\pi$ may be. For example: If we assume $3 \cdot 14159$ to be the value of $\pi$, $\frac{3 \cdot 14159}{50} \times 12.5 = \cdot 0628318 \times 12 \cdot 5 = \frac{\pi}{4} = \frac{3 \cdot 14159}{4} = \cdot 7853975$; and any other hypothetical value of $\pi$ will produce a similar result. But, neither can it be disputed, that if 100 represent the area of a square, $\sqrt{100} = 10$, $=$ diameter of an inscribed circle; Therefore, $\frac{10}{4} = 2 \cdot 5 =$ semi-radius of the circle; and $2 \cdot 5^2 = 6 \cdot 25 = 2\pi =$ circumference of a circle of radius 1, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters. But, $6 \cdot 25 \times 12 \cdot 5 = 78 \cdot 125$, and is equal to 100 times the circular measure of an angle of $45°$ to a

circle of radius 1; which makes the area of every circle exactly equal to $12\frac{1}{2}$ times the area of a square on the semi-radius of the circle. This is my first argument.

Secondly: Referring to the diagram, I may observe, that KN the *sine* of the angle NAK is the perpendicular of the right angled triangle K N A, and equal to A S the semi-radius of the circle X, by construction. But, C D a side of the square C D E F is also the perpendicular of the right angled triangle C D O, and is equal to K L the diameter of the circle X, and KL is equal to 4 (KN); therefore, CD=4 (KN). But, CDO and KNP are similar triangles, by construction, therefore, DO=4 (NP). But, DO= B S, and $\frac{1}{2}$ (B S) = radius of the circle Z, by construction; therefore, the radius of the circle Z = 2 (N P), that is,= twice the base of the right angled triangle K N P, or half the base of the right angled triangle C D O. But, A N the base of the right angled triangle K N A is the radius of the circle X Y Z, by construction; and the sides containing the right angle in the similar triangles C D O and K N P are in the proportion of 3 to 4, by construction. Now, if we take the letters which indicate the circles to represent the arithmetical values of their areas; then, Z is to X Y Z in the proportion of 3 to 4; and X Y Z is to X in the proportion of 3 to 4; therefore, Z : X Y Z : : X Y Z : X, and Z : X : : $3^2 : 4^2$, which may be demonstrated by means of any hypothetical arithmetical value of $\pi$ we may be pleased to adopt, intermediate between 3 and 4; and it follows of necessity, that Z is to X Y Z, and X Y Z to X, in the same proportion as the area of a regular dodecagon or 12 sided regular polygon inscribed in the circle X, to the area of the circumscribing square C D E F. Of these facts any *Geometrician* and *Mathematician* may readily convince himself, and I shall not burden my Letter with the calculations. This is my second argument.

Thirdly: Let the radius of the circle X = 1. Then,

K N the *Natural sine* of the angle N A K $= \frac{1}{2} = \cdot 5$, and K N is the perpendicular of the right angled triangle K N P. But, K N is to N P the base of the triangle K N P, in the proportion of 4 to 3, by construction; therefore, $\frac{3}{4}$ (K N) $= \frac{3 \times \cdot 5}{4} = \cdot 375 =$ N P, and $\frac{5}{4}$ (K N) $= \frac{5 \times \cdot 5}{4} = \cdot 625 =$ K P, the hypothenuse of the triangle K N P; therefore, $KN^2 + NP^2, + KP^2, = (\cdot 5^2 + \cdot 375^2 + \cdot 625^2), = (\cdot 25 + \cdot 140625 + \cdot 390625), = \cdot 78125, = \frac{\pi}{4},$ = the circular measure of the angles N A G and G A B, and is equal to the area of a circle of which the diameter is unity, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters; therefore, $8\,(KN^2 + NP^2 + KP^2) = 8 \times \cdot 78125 = 6\cdot 25 = 2\,\pi =$ circumference of the circle X, and $4\,(KN^2 + NP^2 + KP^2) = 4 \times \cdot 78125 = 3\cdot 125 =$ area of the circle X; and I suspect it would puzzle even you, Sir, to find so extraordinary a phenomenon as a *Geometrician and Mathematician*, who would presume to maintain that the area of a circle of radius one, and the circumference of a circle of diameter one, are not represented by the same Arithmetical symbols, whatever the true value of $\pi$ may be!

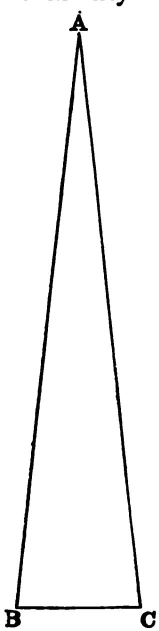
I hardly think my correspondents, "Eminent Mathematician," or Lieut.-Gen. T. Perronet Thompson, would dispute this fact; although I have no doubt the former would tell me, that to base an argument upon it would be to confound linear with square units; and the latter would probably say, that the one represents a line and the other a surface, and are, therefore, irrational quantities, and have no more to do with the ratio of diameter to circumference in a circle, than the price of sugar with the height of the barometer. Now, Sir, I find that if I multiply the area of a square on the radius of a circle, by the area of a circle of which the radius is one, I obtain the same result as if I multiply the area of

the square, by the circumference of a circle of which the diameter is one, and in either case, according to my logic, I obtain the true area of the circle. If you think me wrong, pray say how? This is my third argument.

Fourthly: In the right angled triangle K N P, the perpendicular K N is to the base N P, in the ratio of 4 to 3, by construction; and K N is the *Natural sine* of the angle N A K $= \cdot 5$; therefore, $KN^2 = \cdot 25$, $NP^2 = \cdot 140625$, and $KP^2 = \cdot 390625$, therefore, $\frac{KN^2}{KP^2} = \frac{\cdot 25}{\cdot 390625} = \cdot 64$; and $\frac{NP^2}{KP^2} = \frac{\cdot 140625}{\cdot 390625} = \cdot 36$; therefore, $\frac{KN^2}{KP^2} + \frac{NP^2}{KP^2} = \cdot 64 + \cdot 36 = 1 =$ radius of the circle X; and it follows of necessity, that in every right angled triangle of which the sides containing the right angle are in the ratio of 3 to 4, and the longer of these sides the perpendicular; $\frac{\text{perpendicular}^2}{\text{hypothenuse}^2} + \frac{\text{base}^2}{\text{hypothenuse}^2} =$ radius of a circle of which the circumference $= 2\pi$; therefore, if $\sqrt{\cdot 64}$ and $\sqrt{\cdot 36}$ represent two sides of a right-angled triangle, and contain the right angle; they are in the proportion of 4 to 3, and the hypothenuse of the triangle is equal to the radius of the circle X. This is my fourth argument; and you, Sir, may gnaw away at this bone till you grind your teeth to powder, but you will never succeed in extracting any Orthodox marrow out of it.

Fifthly: I quote the following from the Letter of a gentleman with whom I have had a correspondence, not directly, but through the medium of a mutual friend:— "I must call Mr. Smith's especial attention to what I dare say he is already aware of, that the Mathematical Tables of *Natural sines* is drawn up by calculations which involve no matters of dispute between him and other Mathematicians, and if Mr. Smith doubts the accuracy of the reckoning, it is a simple matter for one who has leisure to review the calculations." Now, in

reply to an argument of this gentleman in the same Letter, derived from one of 60 equal isosceles triangles inscribed in a circle, I admitted that in his calculations there was no error so far as the mere figures were concerned; but his calculations were admittedly based on the assumption that existing Mathematical Tables of *Natural sines* are correct, which I deny. These tables make $\frac{\pi}{3 \times 100} = \frac{\pi}{300}$ the *Natural sine* of an angle of 36 minutes, and so do I! But, as Mathematicians and I are not agreed as to the value of $\pi$, we necessarily differ in our Arithmetical value of the *Natural sine* of an angle of 36 minutes. Existing tables make $\frac{3 \cdot 1416}{300} = \cdot 010472$ the *Natural sine* of an angle of 36 minutes; I make it $\frac{3 \cdot 125}{300} = \cdot 01041666$ to infinity.




Now, let A B C represent one of 25 equal isosceles triangles inscribed in a circle of which the circumference is represented by 360°. Then, $\frac{360°}{25} = 14 \cdot 4 = 14° \; 24'$ is the value of the angle A included by the two equal sides A B and A C, and $\frac{180° - 14° \; 24'}{2} \quad \frac{165° \; 36'}{2} = 82° \; 48'$ is the value of each of the angles B and C; and I admit that the sides of triangles are related to each other as the *sines* of their opposite angles; therefore, B C : A B : : *sine* 14° 24' : *sine* 82° 48'. Now, existing Mathematical Tables give the *Natural sines* of 14° 24' and 82° 48' as 248690 and 992115, which to the third figure are in the ratio of 25 to 100, and this is the true proportion between the *Natural sines* of these two angles. It is true

that on my theory the *Natural sines* of these angles differ from the values of them as given in the Mathematical Tables, but I shall assume, as it is sufficient for my argument, that their values as given in the Tables are correct to the third figure. Well then, $\frac{\cdot 248}{\cdot 992} \times 25 = \cdot 25 \times 25 = 6\cdot 25$, and is *not* equal to the perimeter of the 25 sided regular polygon inscribed in the circle, but equal to the circumference of a circle of radius $1 = 2\,\pi$. Now, $\frac{1}{6}\left(\frac{24}{25}\ 360\right) = \frac{1}{6}\left(\frac{24 \times 360}{25}\right) = \frac{1}{6}\left(\frac{8640}{25}\right) = \frac{345\cdot 6}{6} = 57.6$ = radius of a circle of 360°; and radius $\times\ 2\,\pi =$ circumference in every circle; therefore, $57\cdot 6 \times 6\cdot 25 = 360 =$ the given circumference of the circle. This is my fifth argument.

Sixthly: Let the area of a square $= 100$. Then, the diameter of an inscribed circle $= \sqrt{100} = 10$; and 100 times the circular measure of an angle of 45° to a circle of radius 1, $= 100 \times \frac{\pi}{4} = 100 \times \frac{3\cdot 125}{4} = 100 \times \cdot 78125 = 78\cdot 125 =$ area of the circle. The diameter of a circumscribing circle to the square $= \sqrt{200}$, and the area is equal to twice the area of a circle of which the diameter is 10, and is equal to $25 \times 2\pi = 25 \times 6\cdot 25 = 156\cdot 25$. For, $\frac{1}{2}(\sqrt{200}) = \sqrt{\frac{200}{4}} = \sqrt{50} =$ radius of the circle, and $r^2 \times \pi =$ area in every circle; therefore, $\sqrt{50}^2 \times \pi = 50 \times 3\cdot 125 = 156\cdot 25 =$ area of the circle. Now, if about this circle we circumscribe a square, and about the square circumscribe another circle, the diameter of this circle will be twice $10 = 20$, and the area will be equal to $25 \times 4\pi$ or $100 \times \pi = 4$ times the area of a circle inscribed in a square of which the area $= 100$, and so on we might proceed, *ad infinitum.* But I must observe that these facts are entirely independent of the value of $\pi$, and conse-

quently, for the purpose of demonstration, we may adopt any exact hypothetical value of $\pi$ we please, intermediate between the perimeter of a regular inscribed hexagon, and the perimeter of a circumscribing square, to a circle of which the diameter is unity; that is, intermediate between 3 and 4; and this will be convincing to the mind of any *reflective Mathematician*, that it is an absurdity to suppose that the value of $\pi$ is not Arithmetically expressible with perfect accuracy. This is my sixth argument.

The "*learned fraternity*" who preside over the *interests* of the "British Association for the Advancement of Science" may, in the exercise of their prerogative, refuse to permit me to read a Paper on the question at issue in the Physical Section of the Association; and, assuming, as they do, to be our *guides and authorities* on all questions affecting Mathematical and Geometrical Science, they may ape infallibility, and presume to insult any man who dares to differ from them. Well! well! they may persist in their present course as long as they please, for to me it is now a matter of perfect indifference, and I shall give them no further trouble. All I can say is, if they are wanting in *capacity* to perceive that these incontrovertible truths shiver to atoms the Orthodox "*fandangle*" that the value of $\pi$ can only be expressed Arithmetically by an infinite series, it is not my fault, and I pity their misfortune.

Seventh and lastly: It cannot be disputed that $\frac{\pi}{3}$ expresses the ratio between the circumference of a circle and the perimeter of an inscribed regular hexagon, whatever the Arithmetical value of $\pi$ may be. Now, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters, which makes $\frac{25}{8} = 3{\cdot}125$ the Arithmetical value of $\pi$, the ratio between the circumference of a circle

and the perimeter of an inscribed regular hexagon will be expressed by the Arithmetical fraction $\frac{3{\cdot}125}{3}$, and the two terms of a ratio may be multiplied or divided by the same number without altering the ratio itself; therefore, if we multiply the two terms of the ratio $\frac{3{\cdot}125}{3}$ by 8, we obtain the equivalent ratio $\frac{25}{24}$. Now, $25^2 = 625$, and 24 576, therefore, $\frac{24}{25}(625) = \frac{25}{24}(576) = 600$; therefore, 576 : 600 :: 600 : 625; and it follows of necessity, that $\frac{625}{600}$, $\frac{600}{576}$, $\frac{25}{24}$, and $\frac{3{\cdot}125}{3}$, are equivalent ratios. But, the decimal expression of any and all these Arithmetical fractions is 1·041666 to infinity; therefore, $\frac{1{\cdot}0416}{100} = {\cdot}010416$ is the *Natural sine* of an angle of 36 minutes; and at this point the existing Mathematical Tables of *Natural sines* step in to furnish the proof; for, $\frac{\pi}{3 \times 100}$ $\frac{3{\cdot}1416}{300} = {\cdot}010472$, is the *Natural sine* of an angle of 36 minutes, as given in all Mathematical Tables of *Natural sines.*

Now, let the figure A B C represent one of 600 equal isosceles triangles inscribed in a circle, and let the circumference of the circle be represented by 360.° Then, $\frac{360}{600} = {\cdot}6$, and $\frac{360}{6} = 60$, therefore, $\frac{\frac{6}{10}(60)}{100} = \frac{6 \times 60}{1000} = \frac{360}{1000} = {\cdot}6^2 = {\cdot}36$; and 36 minutes is the value of the angle A included by the two equal sides A B and A C; (and I must here remind you that $\frac{\text{base}^2}{\text{hypothenuse}^2} = {\cdot}36$, in a right angled triangle of which the sides

containing the right-angle are in the proportion of 3 to 4,) therefore, $\frac{180^\circ - 36'}{2} = \frac{179^\circ\ 24'}{2} = 89^\circ\ 42'$ is the value of each of the angles B and C. Now, the *Natural sine* of $36'$ is $\frac{3{\cdot}125}{3 \times 100} = 010416$, and the *Natural sine* of $89^\circ\ 42'$ is, by the existing Mathematical Tables 999986, (and here I may observe that $\cdot 999986$ is so closely approximate to one, the *Natural sine* of an angle of $90^\circ$ to a circle of radius 1, that if not strictly correct, it is so nearly so, that there is no occasion to quarrel with it, and I shall adopt it for the purpose of my argument,) therefore, $\frac{010416}{999986} \times 600 = 6{\cdot}2496$, and is a close approximation to the circumference of a circle of radius $1 = 2\,\pi$; and we may make the approximation as close as we please by extending the number of logarithmic decimals. Well then, by existing Mathematical Tables the *Natural sine* of $36'$ is $\frac{3{\cdot}1416}{3 \times 100}$, $= 010472$ and the *Natural sine* of $89^\circ\ 42'$ is 999986; therefore, $\frac{010472}{999986} \times 600 = 6{\cdot}2832$ $= 2\,\pi$ exactly, on the Orthodox theory; and consequently, 3·142, 3·14, 3·12, or any other Arithmetical quantity intermediate between 3 and 4, might set up to be the true "*paterfamilias*," and shew a fair claim to the title, by means of *Natural sines* of his own manufacture. This is my last argument, and brings us by a very natural process to the mystic number seven.

Now, for the sake of argument, let me admit that these are mere inferences, and entirely dependent upon the correctness of the value given to the *Natural sine* of an angle of 36 minutes; and consequently, that Mathematicians generally, and you in particular, may fairly claim from me some additional and more decided proof, that the logarithmic value I have put upon the *Natural sine* of an angle of 36 minutes is the correct

one. I am not disposed to say a word against the fairness of the claim, and will now proceed to furnish the required proof.

It will not be disputed that the *Natural sine* of an angle of 90° to a circle of radius one, is 1. Now, let the circumference of a circle be 1. Then, $\frac{24}{25}(1) = \frac{24 \times 1}{25}$ $= \cdot 96 =$ perimeter of a regular inscribed hexagon or six sided regular polygon to a circle of circumference 1; and $\frac{1}{\cdot 96}$ and $\frac{\pi}{3}$ are equivalent ratios. For, if we multiply the two terms of the ratio $\frac{1}{\cdot 96}$ by $3 \cdot 125$, we obtain the equivalent ratio $\frac{3 \cdot 125}{3}$, and this is the true Arithmetical expression of the ratio, between the circumference of every circle and the perimeter of an inscribed regular hexagon. Now, my good Sir, if you dispute this fact, I simply make the following request:—Pray furnish me with some other Arithmetical quantity which multiplied by $\cdot 96$ will produce 3, the indisputable Arithmetical value of the perimeter of an inscribed regular hexagon, to a circle of which the diameter is unity. This you must admit to be an utter impossibility; and since $\frac{24}{25}(3 \cdot 125) =$ $\frac{24 \times 3 \cdot 125}{25} = 3$, it follows of necessity, that $\frac{24}{25}$ (circumference) = perimeter of an inscribed regular hexagon to every circle, whatever Arithmetical value we may be pleased to put upon the circumference of the circle.

Well then, $\frac{1}{\cdot 96}$ and $\frac{3 \cdot 125}{3}$ are indisputably equivalent ratios, and both express the ratio between the circumference of a circle and the perimeter of its inscribed regular hexagon. Now, $\frac{1}{\cdot 96 \times 100}, = \frac{1}{96}$, and $\frac{3 \cdot 125}{3 \times 100} =$ $\frac{3 \cdot 125}{300}$, and $\frac{1}{96}$ and $\frac{3 \cdot 125}{300}$ are equivalent ratios, and are fractional expressions of the decimal value of the

*Natural sine* of an angle of 36 minutes; and in finding the decimal expression of the Arithmetical value of the *Natural sine* by these fractions or ratios, there is brought to light some most remarkable Mathematical truths, which have never been discovered by the wisdom of "*Mathematicians and Logicians*," and to which I beg to call your especial attention. For the purpose of bringing these extraordinary Mathematical truths under your notice, it is necessary to perform the operation of working out the sums by long division of simple arithmetic, and set the figures before you.

In the first place, let 1 be divided by 96, the quotient of which is the decimal expression of the *Natural sine* of an angle of 36 minutes.

```
96 ) 1  ( ·010416
     96
     ---
     400
     384
     ---
      160
       96
      ---
      640
      576
      ---
       64
```

The quotient is ·010416, and if extended would be 6 to infinity, with the perpetual remainder 64; therefore, quotient × remainder = ·010416 × 64 = ·666624, and dropping the two last figures, (of which the necessity and the reason for it will be obvious to any first class village school-boy), we obtain the decimal expression of the Arithmetical fraction $\frac{2}{3}$. But, the divisor is 96 and the perpetual remainder 64, and if the remainder be made the numerator of a fraction, and the divisor the de-

nominator, we obtain the Arithmetical fraction $\frac{64}{96}$ which is equivalent to the Arithmetical fraction $\frac{2}{3}$; therefore, if the divisor be made the numerator of a fraction, and twice the perpetual remainder the denominator, we obtain the Arithmetical fraction $\frac{96}{128}$, and this reduced to its lowest terms produces the Arithmetical fraction $\frac{3}{4}$. Now, the fraction $\frac{3}{4}$ expresses the ratio between the perimeter of every regular hexagon inscribed in a circle, and the perimeter of a circumscribing square to the circle. It also expresses the ratio between the area of every regular dodecagon or 12 sided regular polygon inscribed in a circle, and the area of a circumscribing square to the circle. But, referring to the diagram, it also expresses the ratio between the areas of the circles Z and XYZ, and XYZ and X, and between the sides containing the right angle in the triangles K N P and C D O, which play so important a part in the Geometrical figure represented by the diagram. But, you will observe that the first remainder is equal to 4 times the first dividend, the second remainder equal to 4 times the first remainder, and the third remainder equal to 4 times the second, and 16 times the first; therefore, from the first dividend to the perpetual remainder they are in continued proportion, that is, 1 : 4 : : 4 : 16; and 4 : 16 : : 16 : 64; and I may here observe that to this there is but one exception, if the divisor and dividend are in the proportion of 2400 to 25, and to this exception I shall direct your attention presently. Well then, the first remainder is to the second remainder, and the second remainder to the third, as the diameter of a circle to the perimeter of its circumscribing square; and it follows of necessity, that if the first remainder = area of a square on the semi-radius of a circle, the second remainder = area of a square on the radius, and the third remainder =

area of a circumscribing square to the circle. But, you will observe further, that the perpetual remainder is the square of $8 = 64$; and the circular measure of an angle of $45°$ to a circle of radius one $= \frac{\pi}{4}$; therefore, the perpetual remainder multiplied by the circular measure of an angle of $45°$ to a circle of radius one, $= 64 \times \frac{3{\cdot}125}{4} = 64 \times {\cdot}78125$ $= 50 =$ area of a circle of which the diameter is 8; on the theory that 8 circumferences of a circle are exactly equal to 25 diameters; and makes $\left(\sqrt{\frac{\text{area}}{5}} \times \frac{8}{5}\right) =$ radius in every circle; therefore, $\left(\sqrt{\frac{50}{5}} \times \frac{8}{5}\right)$ $= \sqrt{\frac{400}{25}} = \sqrt{16} = 4 =$ radius of the circle. But, radius $\times\ 2\pi =$ circumference in every circle; therefore, $4 \times 2\pi = 4 \times 6{\cdot}25 = 25 =$ circumference of the circle; and it follows of necessity, that $\left(\frac{\text{circumference}}{5}\right) \times \frac{8}{5} = \left(\frac{25}{5} \times \frac{8}{5}\right) = \frac{200}{25} = 8 =$ diameter of the circle; and establishes beyond the possibility of dispute or cavil by any *honest* Mathematician, that 8 circumferences are exactly equal to 25 diameters in every circle.

Again: Let $3{\cdot}125$ be divided by 300, (and $300 : 3{\cdot}125 :: 2400 : 25$), and we obtain the following sum :—

$$
\begin{array}{r}
300\ )\ 3{\cdot}125\ (\ {\cdot}010416 \\
300\phantom{00} \\
\hline
12{\cdot}50\phantom{0} \\
1200\phantom{0} \\
\hline
500 \\
300 \\
\hline
2000 \\
1800 \\
\hline
200 \\
\hline
\end{array}
$$

Now, $\frac{300}{3{\cdot}125}$ and $\frac{96}{1}$ are equivalent ratios; therefore, the quotient is necessarily the same as in the previous example.

If the perpetual remainder and the devisor in this example, be adopted for the numerator and denominator of a fraction, we obtain the arithmetical fraction $\frac{200}{300}$, and this fraction reduced to its lowest terms, gives us the arithmetical fraction $\frac{2}{3}$, as in the previous example. Now, the first remainder is not equal to 4 times the first dividend, but equal to $\frac{24}{25}$ $(\pi \times 4) = \frac{24}{25}$ $(3{\cdot}125 \times 4) = \frac{24}{25}$ $(12{\cdot}5) = \frac{24 \times 12{\cdot}5}{25} = 12$, and is therefore equal to the perimeter of a regular hexagon inscribed in a circle of which the diameter is 4; therefore, in bringing down the figure 5 for a new dividend, we have to affix a point after the 12, making the figure 5 into a decimal quantity, and by doing so, we obtain the arithmetical quantity 12·5, which is equal to 4 times the first dividend; and this is the exception referred to in the last paragraph, and a very remarkable exception it is. Well then, $\pi$ times $12 = 3{\cdot}125 \times 12 = 37{\cdot}5 = \frac{3}{4}$ (50); 4 times 12·5 = 50 = second remainder; and 4 times 50 = 200 = the third and perpetual remainder; and 3·125 : 12·5 :: 12·5 : 50; and 12·5 : 50 :: 50 : 200. But, $\pi$ times diameter = circumference in every circle; therefore, $\pi \times 4 = 3{\cdot}125 \times 4 = 12{\cdot}5 =$ circumference of a circle of which the diameter is 4; and circumference $\times$ semi-radius = area in every circle, therefore, $12{\cdot}5 \times \frac{4}{4} = 12{\cdot}5 \times 1 = 12{\cdot}5 =$ area of a circle of which the diameter is 4; and it follows of necessity, that if the diameter of a circle be 4, the values of the circumference and area are represented by the same arithmetical symbols; and if the circumference of a circle be 4, the values of the diameter and area are represented by the same arithmetical symbols; whatever the arithmetical value of $\pi$ may be; and one hypothetical value of $\pi$ is just as good as another for the purpose of demonstrating these facts. It was the discovery of these

very plain and simple *Mathematical truths*, which at a very early period of my investigation into the ratio of diameter to circumference in a circle, was convincing to my mind, that if an arithmetical quantity (within certain limits) could be found, which would divide into 4 without remainder, it must be the true arithmetical value of $\pi$. Well then, our first dividend in this example is 3·125, the arithmetical value of $\pi$ on the theory that 8 circumferences of a circle are exactly equal to 25 diameters; and our first devisor is 100 times $\frac{24}{25}$th parts of it = 300. Now, $\frac{4}{3 \cdot 125} = 1 \cdot 28$ exactly, and $1 \cdot 28 \times 300 = 384$. But, $\frac{25}{24}(384) = \frac{25 \times 384}{24} = 400$, (that is, = 100 times the first remainder in the previous example, and which you will observe becomes the second dividend); therefore, 384 is the value of the perimeter of a regular hexagon inscribed in a circle of which the circumference is 400; for, $\frac{384}{400}$, $\frac{\cdot 96}{1}$, and $\frac{3}{3 \cdot 125}$ are indisputably equivalent ratios. Now, if we make 2400 and 25 the divisor and dividend, the perpetual remainder will be 1600, which is equal to 25 multiplied by 64. But, 64 is the perpetual remainder in the previous example, and in this example the perpetual remainder is 200. The former is the square of 8, and the latter is the area of a circle of which the radius is 8, on the hypothesis that $\pi = 3.125$; and the consequences, following of necessity from these facts, are very remarkable, and to which I shall now direct your especial attention.

It cannot be disputed, that $\frac{1}{\cdot 96}$ expresses the ratio between the circumference of a circle and the perimeter of an inscribed regular hexagon; and if the two terms of this fraction be multiplied by 3·125, we obtain the equivalent ratio $\frac{3 \cdot 125}{3}$, of which the denominator is the

indisputable value of the perimeter of an inscribed regular hexagon, to a circle of which the diameter is unity. Now, in the fraction $\frac{1}{\cdot 96}$ the numerator is unity, and the denominator is equal to the perimeter of a regular inscribed hexagon to a circle of which the circumference is one; therefore, if the two terms of the ratio $\frac{1}{\cdot 96}$ be multiplied by 3·1416, the approximate arithmetical value of $\pi$ on the Orthodox theory, we obtain the equivalent ratio $\frac{3\cdot 1416}{301,\ 5936}$, and to ascertain the decimal value of the "*Natural sine*" of an angle of 36 minutes by this ratio, we work out the following sum:—

```
301·5936 ) 3·1416    ( ·010416
           3015936
           --------
           12566400
           12063744
           --------
            5026560
            3015936
           --------
           20106240
           18095616
           --------
            2010624
```

The quotient is the same as in the previous examples; and must necessarily be so, since we have not altered the terms of the ratio, between the divisor and dividend. Well then, the first dividend is to the first remainder, as the first remainder to the second remainder; and the first remainder is to the second remainder, as the second to the third and perpetual remainder; that is, they are in continued proportion, as in the first example; and to this there is but one exception when the divisor and dividend are in the proportion of 2400 to 25, and this exception is brought to light by the second example.

The perpetual remainder and divisor in this example are in the proportion of 2 to 3; and the divisor and twice the perpetual remainder are in the proportion of 3 to 4; as in both the previous examples; and establishes all the ratios in connection with the Geometrical figure represented by the diagram, as given in the first example. Now, in the first example the first remainder is the square of 2, the second the square of 4, and the third and perpetual remainder the square of 8; therefore, by cutting off the same number of decimals as there are in the divisor and dividend, the first remainder in this example is equal to the area of a circle of which the radius is 2, on the hypothesis that 3·1416 the first dividend, is the arithmetical value of $\pi$; and on the same hypothesis the second remainder is equal to the area of a circle of which the radius is 4; and the third and perpetual remainder is equal to the area of a circle of which the radius is 8; and to these very remarkable results there is but one exception, when the divisor and dividend are in the proportion of 2400 to 25, and that is, when we make the first dividend the true Arithmetical value of $\pi$.

Now, Sir, you are a Mathematical *elephant*, and have an opportunity of grappling with these *Geometrical and Mathematical truths*. It would indeed be amusing to be behind the scenes, and have the opportunity of observing you pumping your brains for arguments, to prove that they are mere exercises in Mathematical "*gymnastics*," invented by a *Geometrical fly*, that can "*reason wrongly upon no premisses at all*," and have nothing whatever to do with the ratio of diameter to circumference in a circle.

For Astronomical, Nautical, and other purposes we assume the circumference of a circle to be 360°; and $\frac{360}{2\pi}=$

$\frac{360}{6\cdot25} = 57\cdot6$ = radius of the circle; therefore, $\frac{24}{25}\left(\frac{360}{6}\right) = \frac{24 \times 360}{150} = \frac{8640}{150} = 57\cdot6 =$ the chord of an angle of $60°$ and is equal to radius. But, $\frac{60 \times 180}{\pi} = \frac{10800}{3\cdot125} =$ 3456, is the circular measure of an angle of 60°; therefore, $\frac{3456}{60} = 57\cdot6$, is the value of the angle subtended at the centre of a circle by an arc equal to radius. We obtain the same result in the following way:—$360° \times 60 = 21600$, = the circumference of the Earth in geographical miles. $\frac{24}{25}(21600) = \frac{24 \times 21600}{25} = \frac{518400}{25} = 20736 =$ perimeter of a regular inscribed hexagon to a circle of the Earth's circumference in geographical miles; therefore, $\frac{20736}{360} = 57\cdot6$, and is equal to the value of the angle subtended at the centre of the circle by an arc equal to radius. Now, the circular measure of an angle of 45° to a circle of radius one, is $\frac{2\pi}{8} = \frac{\pi}{4}$, and as to this fact there can be no dispute between me and any *Mathematician*. But, differing as we do in our value of $\pi$, we are of course at variance as to the Arithmetical value of the circular measure of the angle, which I make into $\frac{6\cdot25}{8} = \cdot78125$, and is less than the value put upon it by Orthodoxy; and yet, oddly enough, I make the area of a circle of which the circumference is 360 to be greater than it can be made by any possibility, on the Orthodox theory. It cannot be disputed, that circumference $\times$ semi-radius = area in every circle; therefore, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters, $360 \times \frac{57\cdot6}{2} = 360 \times 28\cdot8 = 10368$, = area of a circle of which the circumference is 360; and that this is the true area of the circle, may be readily demonstrated in several ways, but

take the following method of proof. It cannot be disputed, that $r^2 \times \pi =$ area in every circle, and $57{\cdot}6^2 \times 3{\cdot}125 = 331{\cdot}776 \times 3{\cdot}125 = 10368$. But, $\frac{24}{25}(360) = \frac{24 \times 360}{25} = \frac{8640}{25} = 345{\cdot}6$, $=$ 6 times $57{\cdot}6$ $=$ 6 times radius, $=$ perimeter of a regular hexagon or six sided regular polygon inscribed in a circle of which the circumference is 360. Now, if the circumference of a circle $= 345{\cdot}6$, then, $\frac{24}{25}(345{\cdot}6) = \frac{24 \times 345{\cdot}6}{25} = \frac{8294{\cdot}4}{25} = 331{\cdot}776$, $=$ perimeter of an inscribed regular hexagon to the circle. But, $\frac{24}{25}(345{\cdot}6) \times 3{\cdot}125 = 57{\cdot}6^2 \times 3{\cdot}125$, and both $= \frac{20736}{2} = 10368 =$ area of a circle of which the circumference is 360; therefore, the true area of the circle is equal to half the perimeter of a regular hexagon inscribed in a circle of the Earth's circumference in geographical miles. This is my first conclusion.

Again: It cannot be disputed, that 6 (radius $\times$ semiradius) $=$ area of a regular inscribed dodecagon or twelve sided regular polygon to every circle; therefore, $6\left(57{\cdot}6 \times \frac{57.6}{2}\right) = 6\,(57{\cdot}6 \times 28{\cdot}8) = 6\,(1658{\cdot}88) = 9953{\cdot}28$, $=$ area of a regular inscribed dodecagon to a circle of circumference 360; and is equal to $\frac{24}{25}$th parts of the area of the circle, $= \frac{24}{25}(10368) = \frac{24 \times 10368}{25} = \frac{248832}{25} = 9953{\cdot}28$; therefore, $360 : (6 \times 57{\cdot}6) :: 10368 : 9953{\cdot}28$. But, $(6 \times 57{\cdot}6) = 345{\cdot}6$, is the perimeter of a regular inscribed hexagon to a circle of which the circumference is 360; therefore, the circumference of every circle is to the perimeter of its inscribed regular hexagon, as the area of the circle to the area of its inscribed regular dodecagon. But further: Let $345{\cdot}6$ represent the circumference of a circle. Then: $\frac{345{\cdot}6}{2\pi} = \frac{345{\cdot}6}{6{\cdot}25} = \frac{24}{25}(57{\cdot}6)$

$= 55 \cdot 296 =$ radius of the circle, and $r^2 \times \pi =$ area in every circle; therefore, $55 \cdot 296^2, \times \pi = 3057 \cdot 647616 \times 3 \cdot 125 = 9555 \cdot 1488 =$ area of the circle; and $9555 \cdot 1488 : 9953 \cdot 28 :: 9953 \cdot 28 : 10368$; therefore, the area of a circle of which the circumference is equal to the perimeter of a regular inscribed hexagon to a circle of circumference 360, is to the area of a regular inscribed dodecagon to a circle of circumference 360, as the area of the dodecagon to the area of a circle of circumference 360; and since the property of one circle is the property of all circles, it follows of necessity, that this is true of all circles. To produce these results, every other value of $\pi$ but that which makes 8 circumferences of a circle exactly equal to 25 diameters is utterly incompetent. This is my second conclusion.

Again: On the Orthodox theory, 57·2957795 1 &c. is the number of degrees contained in the angle at the centre of a circle subtended by an arc equal to radius; but for the purpose of my argument I may take 57·29578 as a sufficiently close approximation. Now, $360 \times \frac{57 \cdot 29578}{2} = 360 \times 28 \cdot 64789 = 10313 \cdot 2404 =$ area of a circle of circumference 360, on the hypothesis that $\pi = 3 \cdot 14159$; and is less than I make it, notwithstanding that Orthodoxy makes the circular measure of an angle of 45° to a circle of radius one greater than I do. Well then, it is perfectly obvious that Orthodoxy should make the area of the circle greater than I make it, which it does not, and resolves Orthodoxy into an absurdity. But further, the Orthodox value of area cannot be made to harmonize with the relations that I have proved to exist between a circle and its inscribed regular hexagon and inscribed regular dodecagon, which is equally absurd; and I tell you, Sir, without any hesitation, that Orthodoxy would make Geometry itself a mere delusion and a snare.

These facts demonstrate beyond the possibility of dispute or cavil, the utter absurdity of the fanciful and extravagant *vagaries* of Orthodoxy. This is my third conclusion.

Again: The square root of the circular measure of an angle of 45° to a circle of radius 1, multiplied by $2\ \pi$ and divided by 2, is equal to the area of a circle of which the radius is the "*Natural sine*" of an angle of 45°, whatever the arithmetical value of $\pi$ may be. For example: $\sqrt{\frac{\frac{\pi}{4} \times 2\ \pi}{2}} = \sqrt{\frac{\cdot 78125 \times 6\cdot 25}{2}} = \sqrt{\frac{4\cdot 8828125}{2}} =$ $\sqrt{2\cdot 44140625} = 1\cdot 5625$ = area of a circle of which the diameter is $\sqrt{2}$; on the theory that 8 circumferences of a circle are exactly equal to 25 diameters, which makes $\frac{25}{8} = 3\cdot 125$ the arithmetical value of $\pi$; therefore, $\frac{1}{2}(\sqrt{2})$ $= \sqrt{\frac{2}{4}} = \sqrt{\cdot 5}$ = radius of the circle, and is equal to the "*Natural sine*" of an angle of 45°. But it so happens, that any other exact hypothetical value of $\pi$ intermediate between 3 and 4 will produce a similar result; and demonstrates, beyond the possibility of dispute or cavil by any *Geometer and Mathematician*, the absurdity of the Orthodox "*fandangle*," that the value of $\pi$ can only be expressed arithmetically by an infinite series. This is my fourth conclusion.

Again: Referring to the diagram I may observe, that K M L is a right angled triangle of which K L the hypothenuse is equal to twice K M the perpendicular; and he would indeed be a *phenomenon*, who, professing to be a Geometer and Mathematician, would venture to assert, that the foundation of Trigonometrical Science is not traceable to the properties of this particular triangle. Well then, if K L the hypothenuse of the triangle K M L $= 1$, K M the perpendicular = the *Natural sine* of an angle of $30° = \cdot 5$. But, $K L^2 - K M^2 = 1^2 - \cdot 5^2 = 1$

$- \cdot25 = \cdot75 = ML^2$; therefore, $(KL^2 + KM^2 + ML^2)$ $= (1 + \cdot25 + \cdot75) = 2 =$ the sum of the areas of squares on the three sides of the triangle K M L; and is equal to the area of a circumscribing square to a circle of which the diameter is $\sqrt{2}$; and $\frac{1}{2}(\sqrt{2}) = \sqrt{\cdot5} =$ radius, and is the *Natural sine* of an angle of 45°. Now, on the Orthodox theory, 3·1416 is a close approximation to the true arithmetical value of $\pi$; and $\frac{\pi}{4}$ is the circular measure of an angle of 45° to a circle of radius 1; therefore, $\sqrt{\frac{\frac{\pi}{4} \times 2\pi}{2}}$

$$\sqrt{\frac{\cdot7854 \times 6\cdot2832}{2}} = \sqrt{\frac{4\cdot93482528}{2}} = \sqrt{2\cdot46741264} = 1\cdot5708$$

$=$ area of a circle of which the diameter is $\sqrt{2}$, on the hypothesis that $\pi = 3\cdot1416$. This exhibits in a still clearer light, the absurdity of the Orthodox "*fandangle*," that the value of $\pi$ can only be expressed arithmetically by an infinite series. This is my fifth conclusion.

Again: The triangles K M L and K N A are similar triangles, therefore, the angle N A K is an angle of 30°, and the *Natural sine* of this angle is ·5. But, the triangle K N P is a right angled triangle, of which K N and N P the sides containing the right angle are in the proportion of 4 to 3, by construction, and $KN = \cdot5$; therefore, $NP = \frac{3}{4}(\cdot5) = \frac{3 \times \cdot5}{4} = \cdot375$, and K P the hypothenuse, $= \frac{5}{4}(\cdot5) = \frac{5 \times \cdot5}{4} = \cdot625$; therefore, $(KN^2 + NP^2 + KP^2) = (\cdot5^2 + \cdot375^2 + \cdot625^2) = (\cdot25 + \cdot140625 + \cdot390625), = KN^2 \times 3\cdot125 = \cdot25 \times 3\cdot125 = \cdot78125$; and ·78125 is the sum of the areas of squares on the three sides of the triangle K N P, and is therefore equal to the circular measure of an angle of 45° to a circle of radius 1, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters. But, the circular measure

of an angle of $45°$ to a circle of radius one $= \frac{\pi}{4}$, and $\frac{\pi}{4}$ $= \cdot 78125 =$ area of a circle of which the diameter is unity; therefore, since the property of one circle is the property of all circles, it follows of necessity, that the area of every circle is equal to the sum of the areas of squares on the three sides of a right angled triangle of which the sides containing the right angle are in the proportion of 3 to 4, and the longer of these sides the radius of the circle. But, K N $= \cdot 5$ and N P $= \cdot 375$; therefore, the sum of K N and N P $= \cdot 5 + \cdot 375 = \cdot 875$; and the difference of K N and N P $= \cdot 5 - \cdot 375 = \cdot 125$; therefore, the sum and difference of KN and NP $= \cdot 875 + \cdot 125 =$ unity. Now, if $\cdot 875$ and $\cdot 125$ represent two sides of a right angled triangle, and contain the right angle, $\cdot 875^2 + \cdot 125^2 = \cdot 765625 + \cdot 015625 = \cdot 78125$, and is equal to $KN^2 \times 3 \cdot 125 = \frac{\pi}{4} =$ the circular measure of an angle of $45°$ to a circle of radius one, $=$ area of a circle of which the diameter is unity. But $\cdot 875$ and $\cdot 125$ are in the proportion of 7 to 1; and the sum of the squares of the sides which contain the right angle, is equal to the square of the side which subtends the right angle, in every right angled triangle; therefore, since the property of one circle is the property of all circles, it follows of necessity, that the area of every circle is equal to the area of a square on the hypothenuse of a right angled triangle, of which the sides containing the right angle are in the proportion of 7 to 1, and their sum equal to the diameter of the circle. But, $\cdot 765625 - \cdot 015625 = \cdot 75 =$ area of a regular dodecagon inscribed in a circle of which the diameter is unity, and the area of a circumscribing square to a circle of diameter unity is one. But, the perimeter of a regular hexagon inscribed in a circle of diameter unity is 3; and the perimeter of a circumscribing square to the circle is 4; and $3 : 4 :: \cdot 75 : 1$;

therefore, the perimeter of the hexagon is to the perimeter of the circumscribing square, as the area of the dodecagon to the area of the circumscribing square. Now, no other value of $\pi$ but that which makes 8 circumferences exactly equal to 25 diameters in every circle, can be made to harmonize with all these results, and resolves Orthodoxy into a gross absurdity. This is my sixth conclusion.

Again: referring to the diagram I may observe, that B S is the bisecting line of an equilateral triangle inscribed in the circle X, of which K L is the diameter; and B S and K L are in the proportion of 3 to 4, by construction; therefore, if B S and K L represent two sides of a right angled triangle, and contain the right angle, the sum of the areas of squares on the three sides of the triangle is equal to the area of a circle of which K L is the radius; and exactly equal to 4 times the area of the circle X, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters. Now, C D O and K N P are similar triangles, by construction; and D O the base of the triangle C D O = B S, by construction. But, K L = 4 times K N, by construction, and K N is the "*Natural sine*" of the angle N A K = ·5; therefore, if K N = ·5, the sum of the areas of squares on the three sides of the right angled triangle K N P = ·78125. But, C D = K L, = 4 times K N, by construction; therefore C D = 4 times ·5 = 2; D O = $\frac{3}{4}$ (2) = $\frac{3 \times 2}{4} = \frac{6}{4} = 1·5$; and C O = $\frac{5}{4}$ (2) = $\frac{5 \times 2}{4} = \frac{10}{4} = 2·5$; therefore, $(CD^2 + DO^2 + CO^2) = (2^2 + 1·5^2 + 2·5^2) =$ $(4 + 2·25 + 6·25) = 12·5$, is the sum of the areas of squares on the three sides of the triangle C D O; and is equal to sixteen times the sum of the areas of squares on the three sides of the similar triangle K N P; (and C D O

is equal to 16 times K N P as shown in the diagram); therefore, the sum of the areas of squares on the three sides of the right angled triangle CDO multiplied by the square of the "*Natural sine*" of an angle of $30^\circ$, $=12{\cdot}5$ times $KN^2$ $= 12{\cdot}5 \times {\cdot}5^2 = 12{\cdot}5 \times {\cdot}25 = 3{\cdot}125$, is the true arithmetical value of $\pi$; therefore, $12\frac{1}{2}$ times $\frac{\pi}{50} = 12{\cdot}5 \times \frac{3{\cdot}125}{50} = 12{\cdot}5 \times {\cdot}0625 = {\cdot}78125$; and is equal to $12\frac{1}{2}$ times the area of a square on the semi-radius of a circle of which the diameter is unity; $= \frac{\pi}{4}$; $=$ the circular measure of an angle of $45^\circ$ to a circle of radius 1; $=$ area of a circle of which the diameter is unity. This is my seventh and last conclusion.

It was not by any design or effort on my part that I have been led to seven conclusions. It appears to me that these conclusions are all perfectly distinctive in character, and that I could not have consistently omitted any one of them; and I can assure you that with my present knowledge, I cannot add another. I am not naturally superstitious, and yet, I am disposed to agree with you that there is something in the mystic number seven, even in connection with Circle-squaring. Now, Sir, you assume the character of a Mathematical *spider*, and look upon me as a mere Geometrical *fly*, "*utterly destitute of all that distinguishes the reasoning Geometrical investigator from the calculator.*" Well, then, I am now fairly within your grasp, and by entangling me in the web of your *Mathematical infallibility*, you have the opportunity of annihilating me; and I trust you will not be tardy in embracing the opportunity; but I venture to tell you, that before you can accomplish the feat (and you cannot complete your "Budget of Paradoxes" without making the attempt) you must prove: Firstly: That $\frac{24}{25}(3{\cdot}125) = \frac{24 \times 3{\cdot}125}{25} = 3$, is not the arithmetical

value of the perimeter of a regular hexagon inscribed in a circle of which the diameter is unity. And you must prove: Secondly: That $12\frac{1}{2}$ times $\frac{\pi}{50,}$ is not the circular measure of an angle of $45^\circ$ to a circle of radius 1.

In conclusion I may observe, that it is no longer a question of whether the ratio of diameter to circumference in a circle, can or can not be expressed arithmetically with perfect accuracy; or in other words, it is no longer a question of whether the solution of the problem of the "Quadrature of the Circle", is a possibility or an impossibility, for it is "*un fait accompli.*" I say this advisedly, and I do not require your aid, Sir, to enable me to distinguish a demonstration from a fallacy, either in Geometry or Mathematics. But, the subject which has so long been a matter of controversy between me and the mathematical world, has at length resolved itself into a question, and to you a somewhat important question, of whether "*the learned*" in the enlightened age of the 19th century of the Christian era, are, or are not to rank in the future, as equally intolerant with the fanatics who assumed to be "*the learned*" in the dark ages. For the latter, there was the shadow of an excuse: for the former, there is none; and if the "*learned fraternity*" of the present day who assume to be our *guides and authorities* in Mathematical and Geometrical Science, persist in their present course, the only result will be, that another generation will denounce them as worse specimens of humanity, than the persecutors of Galileo.

One word more and I have done: To me, my good Sir, it is now a matter of indifference what course you may be pleased to adopt individually; but I may tell you, that I am impressed with the thought, and "the thoughts we cannot bridle," that silence on your part will be

worse than defeat, and far more damaging to your Mathematical reputation, than any thing you can either write or speak.

I remain,

Sir,

Yours very respectfully,

JAMES SMITH.

IT must not be supposed that in this Letter I have exhausted the subject. I can assure the sincere and earnest enquirer after Geometrical and Mathematical truth, that there is ample scope left him for the exercise of thought, reflection, and ingenuity; and that he will find himself more than repaid for any time and trouble he may devote to the subject. I venture to tell him that he will have no difficulty in discovering innumerable proofs, both Geometrical and Mathematical, of the truth of the theory that 8 circumferences are exactly equal to 25 diameters in every circle. One or two examples by way of hints may serve, form an appropriate addenda to the foregoing Letter, and not be considered out of place.

First example: If circles be circumscribed and inscribed to a square of which the side $= \sqrt{32}$, the diameter of the circumscribing circle is 8, and can be nothing else; and the values of the circumference of the circumscribing circle, and area of the inscribed circle, are represented by the same Arithmetical symbols, whatever the Arithmetical value of $\pi$ may be; therefore, it follows of necessity, that the diameter of the circumscribing circle is to the area of the inscribed circle, as the diameter to the circumference, in the circumscribing circle, whatever the Arithmetical value of $\pi$ may be. But, on no other theory, but that

which makes 8 circumferences of a circle exactly equal to 25 diameters, can the ratio between the diameter of the circumscribing circle and area of the inscribed circle, or the ratio between the diameter and circumference of the circumscribing circle, be in the proportion of 8 to 25; but will necessarily be either greater or less, in exactly the same proportion as we make the value of $\pi$ greater or less than $\frac{25}{8} = 3{\cdot}125$.

Second example: If circles be circumscribed and inscribed to a square of which the perimeter $= \sqrt{32}$, which makes the side of the square $= \sqrt{2}$; the diameter of the circumscribing circle is $\sqrt{4} = 2$, and the area of the circle $= \pi$, and can be nothing else; whatever the Arithmetical value of $\pi$ may be. Now, let the circumference of a circle be represented by $\sqrt{32}$. Then, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters, which makes $\frac{25}{8} = 3{\cdot}125$, the Arithmetical value of $\pi$; $\sqrt{\frac{32}{2\,\pi}} = \sqrt{\frac{32}{6{\cdot}25}} = \sqrt{\frac{32}{6{\cdot}25^2}} = \sqrt{\frac{32}{39{\cdot}0625}}$ $= \sqrt{{\cdot}8192} =$ radius of the circle; and $r^2 \times \pi =$ area in every circle; therefore, $\sqrt{{\cdot}8192}^2 \times \pi = {\cdot}8192 \times 3{\cdot}125$ $= 2{\cdot}56 =$ area of the circle; and it follows of necessity, that on this theory, $\sqrt{\frac{\text{area}}{5} \times \frac{8}{5}} = \sqrt{\frac{2{\cdot}56}{5} \times \frac{8}{5}} = \sqrt{\frac{20{\cdot}48}{25}}$ $= \sqrt{{\cdot}8192} =$ radius of the circle. But, $\pi$ times area of the circle, $= 3{\cdot}125 \times 2{\cdot}56 = 8 =$ diameter of a circumscribing circle to a square of which the side $= \sqrt{32}$. Now, we may establish this conclusion by adopting $3{\cdot}1416$, or $3{\cdot}14159$, or any other hypothetical value of $\pi$ intermediate between 3 and $3{\cdot}2$, on the Orthodox doctrine of approximating theories; but by none but the true value of $\pi$ can we obtain exact results. But further: $\sqrt{2{\cdot}56} = 1{\cdot}6$; and $1{\cdot}6$ may be the radius of a circle.

But, $\sqrt{2{\cdot}56} = 1{\cdot}6$, may also be the perpendicular of a right angled triangle, of which the sides containing the right angle are in the proportion of 3 to 4, and if so, $\frac{3}{4}(1{\cdot}6) = \frac{3 \times 1{\cdot}6}{4} = \frac{4{\cdot}8}{4} = 1{\cdot}2$, will be the base of the triangle; and it will be found by working out the calculations, that the sum of the areas of squares on the three sides of such triangle, is equal to $\pi$ times $1{\cdot}6^2$, $= 3{\cdot}125 \times 2{\cdot}56 = 8$; and therefore equal to the area of a circle of which the radius is $\sqrt{2{\cdot}56}$; and equal to the circumference of a circle of which the diameter is $2{\cdot}56$; and none but the true value of $\pi$ can produce these results.

Third example: Let $\sqrt{1}$ and 7 times $(\sqrt{1})$ represent two sides of a right angled triangle, and contain the right angle. Then, the sum of these two sides $= 8$, and the area of a square on the hypothenuse of the triangle $= 50$, and is equal to the area of a circle of which the diameter is 8; and exactly equal to the sum of the areas of squares on the three sides of a right angled triangle of which the sides containing the right angle are in the proportion of 3 to 4, and the longer of these sides equal to the radius of a circle of which the diameter is 8, $= \frac{8}{2} = 4$. Now, any *Geometer* may describe a circle, and about it circumscribe a square, then divide the sides of the square into two parts in the proportion of 7 to 1, and join the dividing points on each side of the square with the dividing point on an adjacent side, producing a square within the generating square, standing on the inscribed circle. This square is exactly equal to the circle in superficial area, and no *Geometer* can have the slightest difficulty in convincing himself of the fact.

I may simply observe in conclusion, that any Geometrician having satisfied himself of this fact, will

find that he can produce geometrical figures by the score, in demonstration of the truth of the theory, that 8 circumferences are exactly to 25 diameters in every circle.

# APPENDIX.

# APPENDIX A.

BARKELEY HOUSE, SEAFORTH,

LIVERPOOL, 31*st Oct.*, 1864.

SIR,—The following is one of your own proverbs :—"A person of small knowledge is in danger of trying to make his *little* do the work of more." Now, you have clearly worked up your imagination into the belief, and have made some amusing attempts to demonstrate, that a circle-squarer is the greatest of all paradoxers; and, oddly enough, thus furnish in your *own* person a remarkable instance of the truth of your *own* proverb.

In your "Budget of Paradoxes," which still "drags its slow length along," through the columns of the "Athenæum," you are approaching the period (judging from dates) when the writer appeared on the stage of time, acting, as you imagine, a very absurd part in the comedy of circle-squaring. You have been for many years one of the props of the Mathematical drama, and have acquired a world-wide reputation as one of the finest performers on the Mathematical stage. In your great character, that of Champion of Orthodoxy, in which you are especially conspicuous and much admired, you assume infallibility, and denounce all men to be fools who happen to be the victims of circle-squaring propensities. You claim as the reward of your indefatigable exertions (and I think not unreasonably), the right to the dignity of a Mathematical Hierarch, and in the performance of your (self-imposed) duties, have just pronounced, and ordained with all due formality, St. Vitus to be the patron saint of the circle-squarer, who, you tell us (in the style of a genuine disciple of St. Modestus,) leads his votaries a never-ending and unmeaning dance.

It has been said that "the anticipation of a looked-for pleasure is often more delightful than its realization," and such may prove to be my case; yet it is very hard to be held in suspense, as I have been, for months. I am at length relieved, however, by the

thought that I may now fairly anticipate, that ere long you will prove, or at any rate attempt to prove, that I am not an unworthy disciple of your newly-installed patron saint of the circle-squarer.

It is amusing to observe, how great men can sometimes work up their imagination into the belief, that the most extraordinary fancies are truisms. Take yourself as an example, in that particular instance when you hit upon the fancy, that you had discovered an animal which made certain pretensions to be a geometrical reasoner, that could "*reason wrongly upon no premisses at all.*" It does not require a great stretch of imagination to arrive at the conclusion, that some men are fools by nature; that some men are made fools by art; and that some men make fools of themselves; and we may probably next find you hitting upon the fancy, that you have discovered a very remarkable animal of the bipedal genus, that individually fulfils all these conditions.

The person who said, "A man *must* be present when he is shaved," and imagined he had made the discovery of a rule without exception, hit upon a great fancy. The lady who disputed his rule was a shrewd body, and whether ladies have or have not the right to decide controversies about shaving, the lady in question at any rate took off the sharp edge from his fanciful razor; and my namesake, the consulted oracle, was unquestionably right in agreeing with the lady, "that a man *must* be present when he is shaved, is not a *rule*, but a *fact.*"

A truce to all such rodomontade! Let us honestly appeal to the tribunal of simple truth, which will always be found in harmony with plain common sense, whether such a thing does exist in nature, art, or science, as a rule without exception; and if such a rule can be found, whether it can be applied to any useful and practical purpose.

I recently addressed two Letters to the Editor of the "Times," and I cannot say I was surprised at both finding their way into the Editor's waste paper basket. The following is a duplicate of the latter Letter:—

QUEEN HOTEL, HARROGATE,

11*th October*, 1864.

SIR,—I hereby undertake to pay the Sum of One Hundred Pounds to any Public Charity you may name, on your disproving, or finding a Mathematician who can disprove, the following proposition in Mixed Mathematics.

"*Into whatever number of equal arcs the circumference of a circle be divided, if from the value of one of these arcs $\frac{1}{25}$th part be deducted, and the remainder multiplied by the sum of the arcs, the product is equal to the perimeter of a regular hexagon inscribed in the circle.*"

I maintain that this proposition is incontrovertible, and if so, it makes the arithmetical quantity 3·125, the true and exact value of the circumference of a circle of which the diameter is unity. For, the perimeter of an inscribed regular hexagon to a circle of diameter unity, is a known and indisputable quantity, equal to 6 times radius = 3, and every other value of the circumference of a circle of diameter unity, but 3·125, would make the perimeter of the inscribed regular hexagon either greater or less than 3, which is impossible, and therefore absurd. Hence, 1 to 3·125 is the true ratio of diameter to circumference in every circle.*

You have not done me the honour to insert my Letter of the 26th

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* The following is well worth the careful attention of Mathematicians:—It cannot be disputed, that if 8 circumferences of a circle are equal to 25 diameters, $\frac{25}{8} = 3{\cdot}125$ is the arithmetical value of $\pi$. Now, if $\frac{\pi}{6} = x$, and $y =$ the *Natural Sine* of an angle of 30°; $x = \frac{8 \text{ times } \pi}{24} \times y = \frac{25}{24} \times {\cdot}5 = \frac{25 \times {\cdot}5}{24} = \frac{12{\cdot}5}{24} = {\cdot}5208333$ to infinity; and the following is the obvious reason. $\frac{24}{8 \text{ times } \pi} \times \pi =$ perimeter of a regular inscribed hexagon to a circle of which the diameter is unity, which divided by 6 gives the radius of the circle; whatever the arithmetical value of $\pi$ may be. The learned Professor may pass by the reason as some of my correspondents have done, and say, that he can work out the calculations and produce a similar result, with *perfect arithmetical accuracy*, on the theory that $\pi = 3{\cdot}1416$. For example: If $\pi = 3{\cdot}1416$, then, $\frac{\pi}{6} = \frac{3{\cdot}1416}{6} = {\cdot}5236 = x$, therefore, $x = \frac{8 \text{ times } \pi}{24} \times y = \frac{25{\cdot}1328}{24} \times {\cdot}5 = \frac{12{\cdot}5664}{24} = {\cdot}5236$; and consequently, the Professor may possibly tell me as the correspondents referred to have done, that this proves nothing! My reply to this is: The calculations are unquestionably true, and I may admit that the conclusion is plausible; but then, to shew the fallacy that lurks beneath the plausibility, I have simply to put the following question to the learned Professor:—Pray, sir, if your argument be good for anything, what becomes of the Orthodox "*fandangle*," that $\pi$ can only be expressed arithmetically by an infinite series?

ult., in your valuable Journal, but you may yet discover, that it was not unworthy of insertion, even in the columns of the "Times."

I am, Sir,

Yours very respectfully,

JAMES SMITH.

To the Editor of the "Times."

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I gave the Editor references as to my responsibility to fulfil my self-imposed obligation, which in your case is not necessary, for the following reason:—

On hearing from you I will place in the hands of your friend, and if I mistake not, relative—(Capt. B——— S———)—the One Hundred Pounds, to be dispensed in charity in any way you may decide upon, if within One Month from the date of this Letter, you establish an exception to the rule I have laid down for finding the value of the perimeter of a regular hexagon inscribed in a circle, when the circumference of the circle is the given quantity.

I remain, Sir,

Very respectfully yours,

JAMES SMITH.

Professor De Morgan, London.

# APPENDIX B.

## ON THE RELATIONS

EXISTING BETWEEN

## THE DIMENSIONS AND DISTANCES

OF THE

## SUN, MOON, AND EARTH;

A PAPER READ BEFORE THE LITERARY AND PHILOSOPHICAL SOCIETY OF LIVERPOOL, JANUARY 25, 1864.

*(From the Liverpool "Daily Courier," of Jan. 26, 1864.)**

THE seventh ordinary meeting of the Literary and Philosophical Society was held at the Royal Institution, last night, James A. Picton, Esq., in the chair. There was a large attendance.

The following paper was read by James Smith, Esq., "On the relations existing between the Dimensions and Distances of the Sun, Moon, and Earth :"—

MR. PRESIDENT AND GENTLEMEN—

It will be admitted by you, Sir, and by every member of this Society, that the relations existing between the dimensions and dis-

* The Literary and Philosophical Society of Liverpool is a "*British Association for the Advancement of Science,*" in miniature. The "authorities" of the Society announce, that a summary of the proceedings of its meetings will appear in the *Daily Courier.* Knowing the difficulty to any reporter to give a correct summary of a paper of this character, I gave the editor the manuscript, which he inserted at length. This, it appears, was considered a breach of scientific etiquette that could not be permitted with impunity, and consequently, the paper does not appear in the transactions of that learned Association.

tances of the celestial bodies composing our solar system, which is made up of its Sun, with the planets and their satellites revolving about it, and which is only one of the many systems that may, and I believe do exist, in the great starry universe, is a most important subject of inquiry ; and to all who take an interest in Astronomical science, an extremely interesting one.

We are led to believe, or perhaps I should rather say the uninitiated are led to believe, (for Astronomers know better,) that Astronomical science has been brought by the labours of Newton, and those who have succeeded him, to a state of almost absolute perfection ; and yet, the fact is indisputable, that no theory has ever been propounded by which the dimensions and distances of the Sun, Moon, and Earth can be accurately determined.

Now, Ferguson, who wrote about a century ago, and who was a great authority in his day, made the Moon's diameter 2,180 miles, the Earth's diameter 7,970 miles, the Sun's diameter 763,000 miles ; the Moon's distance from the Earth 240,000 miles, and the Sun's distance 82,000,000 miles. These dimensions and distances have subsequently been altered by the common consent of recognised Astronomical authorities ; but for more than half a century it has been considered infallibly true, that 95,365,000 miles was the Sun's distance from the Earth, 237,000 miles the Moon's distance, 2,160 miles the Moon's diameter, 7,912 miles the Earth's diameter, and about 880,000 miles the Sun's diameter.

The Astronomical world, however, has, within the last few months been startled by the announcement, that Astronomers have hitherto been altogether wrong as to the Sun's distance from the Earth. For the full particulars of this announcement, I must refer you to the Letter of Mr. J. R. Hind, the great Astronomer, and Editor of the "Nautical Almanac," which appeared in the *Times* of the 17th of September last, a copy of which I hold in my hand. This Letter commences with the following paragraph :—

"It may occasion surprise to many who are accustomed to read of the precision now attained in the science and practice of Astronomy, when it is stated that there are strong grounds for supposing the generally-received value of that great unit of celestial measures—the mean distance of the Earth from the Sun—to be materially in error, and that, in fact, we are nearer to the central luminary by some 4,000,000 miles than for many years past has been commonly believed. The results of various researches during the last ten years appear,

however, to point to the same conclusion; and, under the impression that the subject may be deemed one of more than scientific interest, I have drawn up a very popular outline of the actual state of our knowledge respecting it, which I now beg to place at your disposal."

The result of Mr. Hind's argument and reasoning is contained in the following paragraph, extracted from his Letter :—

"I subjoin a few of the numerical changes which will follow upon the substitution of M. de Verrier's solar parallax (8″95) for that of Professor Encke, on which reliance has so long been placed. The Earth's mean distance from the Sun becomes 91,328,000 miles, being a reduction of 4,036,000; the circumference of her orbit 599,194,000 miles, being a diminution of 25.360,000; her mean hourly velocity 65,460 miles; the diameter of the Sun 850,100 miles, which is smaller by 38,000. The distances, velocities, and dimensions of all the members of the planetary system, of course, require similar corrections if we wish to express them in miles. In the case of Neptune, the mean distance is diminished by 30 times the amount of correction to that of the Earth, or about 122,000,000 miles."

Now, Sir, whether we adopt the data of Ferguson, the more modern data of Professor Encke, or the newly discovered data of Hind and Le Verrier, no definable relations or proportions would appear to exist between the dimensions and distances of the Sun, Moon, and Earth; and yet, that most important relations actually do exist between the dimensions and distances of these bodies, I shall prove before I conclude this paper. I have thought the time opportune for bringing this subject under the notice of this Society; and a theory, which will be found perfectly harmonious in all its parts, and strictly consistent with mathematical and geometrical truth, is, at any rate, deserving of consideration at the hands of those who claim to be our authorities and guides on this important subject.

I have no doubt that most of you, who have honoured me with your presence this evening, are Geometers and Mathematicians; but probably there are but few of you who have had the time and opportunity to make a practical application of your knowledge to Astronomical science. I shall, therefore, in the first place, call your attention to a theorem in mixed mathematics which you will all understand; and will then make an application of it which, I think, will carry

conviction to your minds as to the truth of the theory I am about to propound.

Well then, as Geometers and Mathematicians, it will be conceivable to you that a circle (which we will call A) may exist, of which the diameter shall be represented by 2000—it may be 2000 inches, feet, yards, or miles. It will also be conceivable to you that another circle may exist (which we will call B), of which the diameter shall be equal to 4 times the diameter of the circle A—that is, = 8000, and placed at a distance from A, which shall be represented by 120 times the diameter of circle A—that is, by 240·000. And, further: It will also be conceivable to you that a third circle may exist (which we will call C), of which the diameter shall be equal to 120 times the diameter of the circle B—that is, = 120 (8000) = 960,000, (= 4 times the distance of the circle A from the circle B) and placed at a distance from B, which shall be represented by 120 times the diameter of circle C—that is, by 120 (960,000) = 115,200,000.

Now, as the property of one circle is the property of all circles, the ratio of diameter to circumference in these three circles must be the same, and on the hypothesis that 8 circumferences of a circle are equal to 25 diameters, $\frac{25}{8} = 3·125$ is the arithmetical value of $\pi$; and $\pi$, as you all know, is adopted by Mathematicians to represent the circumference of a circle of which the diameter is unity; therefore, on this hypothesis, the ratio of diameter to circumference in a circle is as 1 to 3·125.

Then:

A's distance from B, 240,000, ... ... ... ... Log. 5·380211
A's circumference, 2000 x 3·125 = 6,250, ... ... Log. 3·795880

Log. ratio 1·584331

And the natural number corresponding to the Logarithm

$$\text{ratio} = \frac{\text{A's distance}}{\text{A's circumference}} = \frac{240·000}{6·250} = 38·4.$$

B's circumference, 8000 x 3·125 = 25,000 ... ... Log. 4·397940

This is the Logarithm of C's diameter = 120 (B's diameter) = 120 (8000) = 960,000 = 38·4 (B's circumference) ... ... ... } ... Log. 5·982271

$$\frac{\text{B's circumference}}{\text{B's diameter}} = \frac{25,000}{8000} = 3·125, \ldots \quad \ldots \quad \ldots \text{ Log. } 0·494850$$

This is the Logarithm of C's circumference = 3·125 (C's diameter) = 960,000 x 3·125 = 3,000,000 } Log. 6·477121

$\frac{\text{A's distance}}{\text{A's circumference}} = \frac{240{,}000}{6{,}250}, = 38{\cdot}4,$ ... ... Log. 1·584331

Log. 8·061452

And this is the Logarithm of 115,200,000, C's distance from B; and the ratio of diameter to circumference in the three circles, A, B, and C, is as 1 to 3·125.

Now, by hypothesis, let the value of $\pi$ be 3·14.

Then:

A's distance from B, 240,000, ... ... ... ... Log. 5·380211

A's circumference, 2000 × 3·14 = 6,280, ... ... Log. 3·797960

Log. ratio 1·582251

B's circumference, 8000 × 3·14 = 25,120, ... ... Log. 4·400020

This is the Logarithm of 960,000 = C's diameter, ... Log. 5·982271

$\frac{\text{B's circumference}}{\text{B's diameter}} = \frac{25{,}120}{8000}$ 3·14, ... ... ... Log. 0·496930

Log. 6·479201

Log. ratio 1·582251

Log. 8·061452

This is the Logarithm of C's distance from B, 115,200,000; and on this hypothesis, the ratio of diameter to circumference in the three circles, A, B, and C, is as 1 to 3·14.

But $\frac{\text{A's distance}}{\text{A's circumference}} = \frac{240{,}000}{6{,}280} = 38{\cdot}21$, &c., and the Logarithm of this number is 1·582177 &c., and is less than the Logarithm ratio. But further: by extending the decimals, we may make $\frac{\text{A's distance}}{\text{A's circumference}} \times$ B's circumference, approximate as close as we please to C's diameter, but we can never by any possibility reach it.

Now, we may hypothetically assume the value of $\pi$ to be 3·1416, 3·1415926, 3·12, or any other quantity intermediate between the perimeters of an equilateral inscribed hexagon and circumscribed square to a circle of which the diameter is unity—that is, between three and four—and by this method of calculation arrive at the true value of C's diameter and C's distance from B; but by no value of $\pi$ but that which makes 8 circumferences of a circle equal to 25 diameters can we make $\frac{\text{A's distance}}{\text{A's circumference}}$ exactly equal to the Log. ratio; and this is of itself a strong presumptive evidence that 1 to 3·125 is

the true ratio of diameter to circumference in a circle, if not an absolute proof.

Now, those who assume to be our Mathematical authorities deny that this is the true ratio of diameter to circumference in a circle, and audaciously assert that I assume this without the shadow of a proof; and fancy, forsooth, that by putting such questions as the following they prove the impossibility of there being a definable numerical ratio between the diameter and circumference of a circle:—Divide 1 by 3, Sir, without a remainder? Tell us the diagonal of a square? Double a cube and tell us its contents? Such a course of procedure is not more absurd than if these wiseacres were to pretend to tell the price of cotton in the Liverpool market by the height of the barometer at Greenwich.

Now, Sir, suppose these gentlemen to put the following question to me:—Can you double a cube, and by means of it demonstrate the ratio of diameter to circumference in a circle? This would be a sensible question, and would demand from me a plain and distinct answer; and I shall now assume that you, Sir, have put this question to me, and I will proceed to answer it.

Well then, every Geometer and Mathematician knows that if the diameter of a circle be 4, the arithmetical symbols which represent the value of the circumference, also represent the value of its area; and every Geometer and Mathematician also knows, that if the circumference of a circle be 4, the arithmetical symbols which represent the value of the diameter of the circle, also represent the value of its area.

Now, twice the cube of $4 = (4 \times 4 \times 4 \times 2) = 128$, and 100th part of $128 = \frac{128}{100} = 1{\cdot}28$. Then:

| | |
|---|---|
| The Log. of number 4 is | 0·60206 |
| The Log. of number 1·28 is | 0·10721 |
| Log. ratio | 0·49485 |

And the natural number corresponding to this Logarithm is 3·125, and $1{\cdot}28 \times 3{\cdot}125 = 4$, therefore $\frac{1{\cdot}28}{4}$ and $\frac{1}{3{\cdot}125}$ are equivalent ratios, and both express the ratio of diameter to circumference in a circle, and makes 3·125 the numerical value of $\pi$.

But further: $\frac{\pi}{4}$ represents the area of a circle of which the diameter is unity; therefore, $\frac{3{\cdot}125}{4} = {\cdot}78125$ is the area of a circle

of which the diameter is unity; and ·78125 × 1·28 = unity; therefore, $\frac{·78125}{1}$ and $\frac{1}{1·28}$ are equivalent ratios, and both express the ratio between the area of every circle, and the area of a square circumscribed about it. The Mathematical and Astronomical authorities who have attempted to write me down a fool, may, and probably will, continue to make a mystery of their craft, and jealously guard it; but I venture to tell them that they may fiddle upon the strings of their mathematical ingenuity until their heads ache, but they will never succeed in controverting these mathematical and geometrical truths.

This brings me to the direct and immediate object of my Paper: the relations existing between the dimensions and distances of the Sun, Moon, and Earth; and it is necessary to call your attention to the fact, that I am perfectly aware the planets are not continuously at an exact distance from the Sun: thus, the Earth is at one time further from and at another time nearer to the Sun than its mean distance; but these deviations from the mean distance compensate each other in an entire revolution of the Earth about the Sun, and consequently the Earth occupies the same relative position to the Sun at the end of a revolution that it did at the commencement of it. It is not my object or intention to go into these deviations and their causes this evening, and you will therefore please bear in mind that my observations are confined to the mean distances of the Sun, Moon, and Earth from each other.

Well then, the Astronomer Royal, Mr. Airy, makes the equatorial diameter of the Earth in miles 7925·648, and the polar diameter in miles 7899·170. The equatorial circumference being, as he says, a little less than 25,000—accurately 24,899. (See *Encyclopœdia Metrop.* Art., Figure of the Earth.) This makes the Earth's mean circumference 24,857 miles, which, divided by 3·1416, the Astronomer's value of $\pi$, makes 7912·4 miles to be the Earth's mean diameter, approximately; and these dimensions of the Earth are adopted by Astronomers generally. Now, it is perfectly obvious that if the Astronomer's value of $\pi$ be false, 7912·4 miles cannot be the Earth's true diameter; and it is edually obvious that if 3·1416 be greater than the true value of $\pi$, the diameter of the Earth must be greater than 7,912 miles. I hardly think any of you will venture to dispute the possibility of a slight error (only about 100 miles in 25,000) in the equatorial circumference of the Earth as shown by the

Astronomer Royal, when you consider the difficulties and consequent liability to error by which the value of the Earth's circumference is ascertained, although it must be admitted that Astronomers obtain this by what they call actual measurement, and assume it to be absolutely and infallibly true. I maintain, on the contrary, that the Astronomer's value of $\pi$ is erroneous, and that it is an utter impossibility to ascertain the true dimensions of such bodies as the Sun, Moon, and Earth, truly and exactly, by any other than geometrical and mathematical measurement, however approximately we may arrive at the Earth's dimensions by actual measurement, even by a false value of $\pi$.

Now, I make the Earth's circumference by geometrical and mathematical measurement to be 25,000 miles, and $\frac{25000}{3 \cdot 125} = 8{,}000$ miles to be the Earth's diameter.* That these are the ***true dimensions*** of the Earth, may be proved in the following way:—For astronomical and nautical purposes we divide the equatorial circumference of the Earth into 360°; therefore, a degree of longitude, on the equator, is exactly equal to 1-360th part of the Earth's equatorial circumference. We make a day and night to consist of 24 hours of 60 minutes each; therefore, $\frac{24 \times 60}{360} = \frac{1440}{360} = 4$ minutes. Now, it is conceivable, and it may be supposed possible, that a ship might start from a point on the equator, and sail round it to the same point again in a day of 24 hours; it is obvious under these supposed circumstances that she would make a degree of longitude every four minutes. As the degrees of longitude diminish from the equator to the poles, it is also conceivable, that a ship might sail round a circle of the Earth so near to one of the poles as to actually make the voyage round the circle in exactly 24 hours, in which case she would make one degree of longitude in every four minutes, just as in the case supposed to occur on the equator.

Now, if the value of a degree of longitude on the equator be ascertained on a false value of the Earth's circumference, which it

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* The heavenly bodies were not called into existence by the Creator, and their dimensions arranged, on a measure of value based on English miles. I must therefore not be understood as employing the word literally, but for want of a better. The Creator in His wisdom has made the Sun and all the planetary bodies with diameter to equatorial circumference in the ratio of 8000 to 25,000, and for all astronomical purposes, it is sufficient for us to know this fact.

actually is, it necessarily follows that the navigator as he sails from the equator towards the poles must find a regularly increasing error between time by observation and time by chronometer; and the fact is so, so that in high latitudes, say from 60° to 70°, he finds himself unable to tell with certainty the position of his ship within 20 to 25 miles. This would be impossible if the degrees of longitude were calculated upon the true circumference of the earth at the equator; for, since the Earth's circumference is divided into 360°, giving 4 minutes of time to every degree of longitude, time by observation and time by chronometer on any part of the Earth's surface must necessarily be in harmony; that is, assuming the chronometer to keep true time, and the circumstances favourable for making the observation. Now, I have the authority of several navigators, men of intellect, education, general knowledge, and nautical experience, for saying, that allowing for the small difference between the true and supposed equatorial circumference, and calculating the degrees of longitude accordingly, rectifies the error arrived at by the calculations made from existing tables. It is only a few days ago that I had a conversation on this subject with Capt. Judkins, the Commodore of the Cunard line of steamers, a gentleman well known to all of you, when he told me that in the latitudes of 40° to 50°, there is a difference of from 10 to 15 miles between time by observation and time by chronometer, confirming precisely the truth of my theory.

Now, this makes the Earth's diameter to be $\frac{25000}{3{\cdot}125}$ = 8,000 miles, and the Moon's diameter is to the Earth's diameter as the Earth's diameter to the perimeter of a square, circumscribed about a circle of the Earth's diameter; and 4 times 8,000 = 32,000, = the perimeter of the circumscribed square; therefore, as 32,000 : 8,000 : : 8,000 : 2,000, and 2,000 miles is the Moon's diameter, and is therefore exactly equal to ¼th part of the Earth's diameter. The Moon's distance from the Earth is equal to 30 times the Earth's diameter, and is therefore equal to 20 times the perimeter of an equilateral triangle, of which the sides are equal to the Earth's radius, or 10 times the perimeter of an equilateral inscribed hexagon to a circle of the Earth's diameter, = 60 times the Earth's radius, = 60 times 4,000, = 240,000 miles. (*See Diagram, page* 91.)

Then :

Moon's distance from the Earth, 240,000 miles, ... Log. 5·380211

Moon's circumference = 2,000 x $\pi$

= 2,000 x 3·125 = 6,250 miles,... ... Log. 3·795880

Log. ratio, 1·584331

And :

$\frac{\text{Moon's distance}}{\text{Moon's circumference}} \frac{240,000}{6,250} = 38{\cdot}4.$ And this is the natural number corresponding to the Log. ratio.

Now, as it is an utter impossibility to obtain this result, either by increasing or diminishing the Moon's distance, the Moon's diameter, or the value of $\pi$ separately; or by altering the value of $\pi$ and conjointly or separately increasing or diminishing the Moon's diameter and distance; or by retaining the value of $\pi$ and in any way altering the proportions between the Earth's diameter, Moon's diameter, and the Moon's distance; it necessarily follows, that these are the true proportional relations existing between the dimensions and distance of the Moon and Earth, and *per se* establishes the true numerical value of $\pi$ to be 3·125.

But, when we extend our inquiry to the dimensions of the Sun, its distance from our own planet, and its relations with our own planet's satellite the Moon, these relations become marvellous, and furnish us with a demonstration of the littleness of man's wisdom, and the superlative wisdom of the great Creator.

Well then, on the foregoing data we find :

First: As Moon's diameter : Earth's diameter : : Moon's distance: Sun's diameter; or, as 2,000 : 8,000 : : 240,000 : 960,000. Second: As Moon's diameter : Moon's distance : : Earth's diameter : Sun's diameter; or, as 2,000 : 240,000 : : 8,000 : 960,000. Third: As Earth's diameter : Sun's diameter : : Sun's diameter : Sun's distance from the Earth; or, as 8,000 : 960,000 : : 960,000 : 115,200,000; therefore, 120 times Moon's diameter = Moon's distance from the Earth; 120 times Earth's diameter = Sun's diameter; and 120 times Sun's diameter = Sun's distance from the Earth; therefore, Moon's distance from the Earth = 60 times the Earth's radius.*

* "The diameter of the Earth being determined (and the writer explains how this is done), its radius is known, and we are prepared to explain the process by which the Moon's distance may be found.

"Let us locate, in imagination, two observers at distant points on the same great circle of the Earth, each prepared to measure the angular distance

Thus, we have Moon's circumference : Moon's distance : : Earth's circumference : Sun's diameter. Or, as 6,250 : 240,000 : : 25,000 : 960,000. And, Moon's distance × Sun's circumference = Sun's distance × Moon's circumference; therefore,

$$\frac{\text{Moon's distance} \times \text{Sun's circumference}}{\text{Moon's circumference}} = \frac{240{,}000 \times (960{,}000 \times 3{,}125)}{2{,}000 \times 3{,}125}$$

$$= \frac{240{,}000 \times 3{\cdot}000{,}000}{6{,}250} = \frac{720{,}000{,}000{,}000}{6{,}250.} = 115{,}200{,}000 \text{ miles} =$$

Sun's distance from the Earth.

Proof:

| | |
|---|---|
| Moon's distance from the Earth, 240,000 miles, ... | Log. 5·380211 |
| Moon's circumference = 2,000 × $\pi$ = 2,000 × 3,125 = 6,250 miles, ... ... ... | Log. 3·795 880 |
| | Log. ratio, 1·584331 |
| Earth's circumference 8,000 × $\pi$ = 8,000 × 3,125 = 25,000 miles, ... ... | Log. 4·397940 |
| Sun's diameter, 960,000 miles, ... ... ... | Log. 5·982271 |

at which the Moon appears from the zenith point of each station; but the zenith of any place is the point in which the Earth's radius prolonged reaches the heavens; the angular distance of the Moon from the zenith will exhibit precisely the inclination of the visual ray drawn to the Moon's centre with the Earth's radius drawn to the place of observation; the zenith distances being observed at each station, the observers knowing that part of the great circle of the Earth by which their stations are separated, come together, compare observations, and construct a figure composed of four lines. Two of these are the radii of the Earth drawn to the points of observation. These may be laid down under their proper angle, drawing from their extremities two lines, forming, with the radii, angles equal to the Moon's measured zenith distances. These represent the visual rays drawn to the Moon; they meet in a point which determines their length, and if the figure be constructed accurately, it will be found that either of these lines is about sixty times longer than the radius of the Earth, or the Mcon's distance is about 240,000 miles.—*The Orbs of Heaven, or the Planetary and Stellar Worlds.* By O. M. Mitchell, A.M., Director of the Cincinnati Observatory. Page 76.

"The Astronomer Royal makes the Earth's equatorial circumference 24,899 miles. Now, on this data, $\frac{24{\cdot}899}{2\pi} = \frac{24{\cdot}899}{6{\cdot}25} = 3983{\cdot}84$ miles = Earth's radius, which, multiplied by 60, gives 239,030 as the Moon's distance. But, remembering that He who called these bodies into existence, did not create them on a scale of English miles:—Can any reasonable man have a doubt that the Earth's diameter is 4,000, and Moon's distance 240,000, which makes the Earth's circumference 25,000, whether we denominate these arithmetical quantities by the term miles or by any other name."

$\frac{\text{Earth's circumference}}{\text{Earth's diameter,}} = \frac{25{,}000}{8{,}000} = 3{\cdot}125,$ ... ... Log. 0·494850

Sun's circumference $= 960{,}000 \times \pi = 960{,}000 \times 3{\cdot}125 = 3{,}000{,}000$ miles, ... ... Log. 6·477121

$\frac{\text{Moon's distance,}}{\text{Moon's circumference}} = \frac{240{,}000}{6{,}250} = 38{\cdot}4,$ ... ... Log. 1·584331

Log. 8·061452

which is the Logarithm corresponding to the natural number 115,200,000 miles, the Sun's distance from the Earth; and you will observe that the Logarithm of $\frac{\text{Moon's distance}}{\text{Moon's circumference}}$ and the Log. ratio are the same. This is perfectly unique, and no other proportions between the dimensions and distances of the Sun, Moon, and Earth can by any possibility be devised which can produce the same results; nor can these results be obtained by any other value of $\pi$ but that which makes 8 circumferences of a circle equal to 25 diameters; and demonstrates beyond the possibility of dispute or cavil by any honest Mathematician and Astronomer, that these are the true proportional relations existing between the Sun, Moon, and Earth, and establishes the true numerical value of $\pi$ to be 3·125.

I must now call the attention of the Society to some very curious facts. Having obtained the true dimensions of the Moon and Earth, and their true mean distance from each other, we may adopt any hypothetical value of $\pi$ intermediate between the perimeter of an inscribed equilateral hexagon and circumscribed square to a circle, of which the diameter is unity, that is, between 3 and 4, and ascertain the true diameter of the Sun, and its true distance from the Earth.

For example: By hypothesis, let the value of $\pi$ be 3·12.

Then:

Moon's distance from the Earth, 240,000 miles, ... Log. 5·380211

Moon's circumference, $2{,}000 \times 3{\cdot}12 = 6{,}240$ miles, ... Log. 3·795185

Log. ratio 1·585026

Earth's circumference, $8{,}000 \times 3{\cdot}12 = 24{,}960$ miles,... Log. 4·397245

This is the Logarithm of 960,000 miles, the Sun's diameter ... ... ... ... ... Log. 5·982271

$$\frac{\text{Earth's circumference,}}{\text{Earth's diameter,}} = \frac{24.960}{8,000} = 3 \cdot 12 \text{ miles,} \qquad \ldots \text{ Log. } 0 \cdot 494155$$

Log. 6·476426
Log. ratio, 1·585026

Log. 8·061452

and the natural number corresponding to this Logarithm is 115,200,000, the Sun's distance from the Earth.

You will observe that we obtain the Logarithm of the true diameter of the Sun, the Logarithms of the diminished circumferences of the Moon and Earth compensating each other. You will also observe that by adding the Log. ratio to the sum of the Logarithms of the Sun's diameter and $\frac{\text{Earth's circumference}}{\text{Earth's diameter}}$, we obtain as the final result the Logarithm of the Sun's distance from the Earth, the intermediate differences again compensating each other. But, if instead of adding the Log. ratio to the Logarithm 6·476426, we added (as we should do) the Logarithm of

$$\frac{\text{Moon's distance}}{\text{Moon's circumference}} = \frac{240 \cdot 000}{6 \cdot 240} = 38 \cdot 46, \text{ \&c.,}$$

the Logarithm of which is 1·585009 &c., we should fail to obtain as the final result the distance of the Sun from the Earth; and although by extending the decimals, we may make $\frac{\text{Moon's distance}}{\text{Moon's circumference}} \times$ Earth's circumference, approximate as close as we please to Sun's diameter, we can never by any possibility reach it.

Now, Mathematicians tell us that the numerical value of $\pi$ is greater than 3·1415926, and less than 3·1416, and assert that it is somewhere between the two, but cannot be expressed in figures with perfect exactness; but it so happens, that we may adopt either of these values of $\pi$, and produce the same result as in the foregoing example; and this proves that one value of $\pi$ is as good as another for the purpose of demonstrating many mathematical and geometrical truths, but it at the same time demonstrates the absurdity of supposing that the numerical value of $\pi$ is, or can be, an incommensurable quantity.

I must not omit to call the attention of the Society to the fact, that having established the true value of $\pi$ to be 3·125, and the true proportions between the dimensions and distance of the Moon and Earth, we may increase or diminish the value of the Earth's diameter without disturbing or affecting the truth or force of my theory. For example:

By hypothesis, let the Earth's diameter be less than 8,000 miles, say 7,920 miles.

Then:

$3\frac{1}{8}$ (7,920) = 24·750 = Earth's circumference.

$\frac{1}{4}$ (7,920) = 1,980 miles = Moon's diameter.

$3\frac{1}{8}$ (1,980) = 6187·5 miles = Moon's circumference.

3 (7,920) = 23,760 miles = perimeter of an inscribed regular hexagon to a circle of the Earth's diameter.

10 (23,760) = 237,600 miles = Moon's distance from the Earth.

4 (237,600) = 950,400 miles = Sun's diameter.

$3\frac{1}{8}$ (950,400) = 2,970,000 miles = Sun's circumference.

Therefore, on this hypothesis,

$$\frac{\text{Moon's distance} \times \text{Sun's circumference}}{\text{Moon's circumference}} = \frac{237{,}600 \times 2{,}970{,}000}{6187{\cdot}5} =$$

114,048,000 miles = the Sun's distance from the Earth, the Logarithm of which is 8·057087.

Now:

| | |
|---|---|
| Moon's distance, 237,600 miles, ... ... ... | Log. 5·375846 |
| Moon's circumference, 6187·5 miles, ... ... ... | Log. 3·791515 |
| | Log. ratio 1·584331 |
| Earth's circumference, 7,920 × 3·125 = 24,750 miles, | Log. 4·393575 |
| Sun's diameter, 950,400 miles, ... ... ... ... | Log. 5·977906 |
| $\frac{\text{Earth's circumference}}{\text{Earth's diameter}} = \frac{24{,}750}{7{,}920} = 3{\cdot}125$ ... ... | Log. 0·494850 |
| Sun's circumference, 2,970,000 miles, ... ... ... | Log. 6·472756 |
| $\frac{\text{Moon's distance}}{\text{Moon's circumference}} = \frac{237{,}600}{6187{\cdot}5} = 38{\cdot}4$ ... ... | Log. 1·584331 |
| | Log. 8·057087 |

And this is the Logarithm of 114,048,000 miles, the Sun's distance from the Earth, and the harmony and consistency of my proportional theory is not in the slightest degree affected.

Now, it will be obvious to you, Sir, and to every member of this Society, that if we are unable to ascertain truly and exactly the dimensions of our own planet the Earth, we must for ever remain ignorant of the true dimensions of the other celestial bodies composing our solar system, for this is the source from which all our information on this subject is derived, and the base upon which all our knowledge rests. And it is equally certain that a very slight error in the Earth's diameter and distance from the Moon, leads to errors

of enormous magnitude in the dimensions and distance of the Sun, and of the still more distant bodies—the planets. I have called your attention to the fact that Mr. Hind diminishes the distance of Neptune by no less than 122,000,000 miles, in consequence of the supposed discovery of an error in the solar parallax. A quantity not measured by degrees or minutes, but by parts of a second.

The importance of first ascertaining the Earth's diameter, circumference, and distance from the Moon will, therefore, be apparent; and it is worth while to consider how far my data is inconsistent with Orthodoxy. If I take the datum of Ferguson, as to the Moon's distance from the Earth—and he was certainly considered Orthodox in his day—we are agreed in making it to be 240,000 miles; and as regards the Earth's diameter, I am only heterodox thirty miles in 8,000, or something less than a broker's commission. If I take the data of the Astronomer Royal, we differ 101 miles in 25,000, as to the Earth's circumference, and if, upon his data, I ascertain the Earth's diameter by the true value of $\pi$, I make it $\frac{24899}{3 \cdot 125} = 7967 \cdot 68$ miles, and again much less than a broker's commission covers our differences.* Under these circumstances, I think you will agree with me, that Astronomers will not be justified or excused if they reject and despise, and consequently refuse their consideration to a theory which is thoroughly harmonious in all its parts, and strictly consistent with mathematical and geometrical truth; while at the same time their own theory rests upon a base that would for ever render absolute truth an impossibility, namely, the incommensurability which they assert to exist between the diameter and circumference of a circle.†

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* Many of my hearers were commercial gentlemen, and to them these similes would be perfectly intelligible, and under the circumstances may not be considered out of place.

† "*The Principia and the Bible: a Critique and an Argument, with an Appendix on the Scale of the Universe.*" By J. H. Macdonald.—The object of this work is to prove the Biblical Physics to be true, in opposition to the concession made to Infidelity, that in matters of Natural Philosophy the Bible is at fault. The author has very ably performed his task, aud clearly points out the inconsistencies of some of our Astronomical authorities with themselves, and of many of them with each other. Without defining his own, the author exposes some of the absurdities of received Astronomical theories with remarkable distinctness and simplicity. A perusal of the work will repay every sincere and unprejudiced inquirer after truth.

Well then, the readers of Mr. Hind's Letter will observe that the reason assigned by him for the diminished distance of the Earth from the great central luminary is, that from certain elaborate calculations made by Le Verrier, based upon the transits of Venus across the Sun's disc in 1761 and 1769, they are led to doubt the accuracy of the solar parallax as worked out by Professor Encke, which he made to be 8″57, or, for the Earth's distance from the Sun, 95,365,000 miles; and Mr. Hind, adopting the calculations of Le Verrier, who makes the solar parallax 8″95, consequently reduces the Sun's distance to 91,348,000 miles.

Now, in attempting to ascertain the solar parallax by the transits of Venus, the diameter of the Earth is an essential element, and so long as Astronomers shall continue to persist that they are infallibly correct as to the Earth's diameter, so long must they be content to remain ignorant of the true distance of the Sun from the Earth. The next transits of Venus, in 1874 and 1882, will no doubt be looked forward to by Astronomers with great interest; but I venture to tell them, that unless in the meantime they make themselves better acquainted than they are at present with the dimensions of the Earth and Moon, they will not find themselves one whit the wiser for the transits of Venus, as to the true dimensions, distance, and density of our great central luminary.

I had much more to say had the limits of a Paper permitted it, but I may on some future opportunity, again direct the attention of the Society to this interesting and important subject.

A short discussion followed the reading of the Paper, and several members stated objections to some of Mr. Smith's views.

The Rev. JOSHUA JONES objected that if Mr. Smith, instead of confining his Logarithms to six places of decimals, had extended them to twenty places, he would have found the result different. He also objected that Mr. Smith had assumed that the heavenly bodies were always working in a true circle, which he held was not true, for they moved in ellipses.

Mr. BEHREND replied that, supposing the Logarithmic decimals were extended, it was really no argument against the proofs which Mr. Smith had given.

After some remarks from Dr. Commins, Mr. Weightman, and Mr. J. M'Farlane Grey,

The CHAIRMAN said that, notwithstanding the elaborate calculations of Mr. Smith, which were very ingenious, they were not one

bit nearer the matter than they were before. He assumed that if the heavenly bodies did exist at these supposed distances at which he made them out, there would be a great harmony and beauty in the solar system ; but whether harmony did exist or not was a question that had to be proved by observation. He, however, gave Mr. Smith credit for the ingenuity and ability with which he had treated his subject.

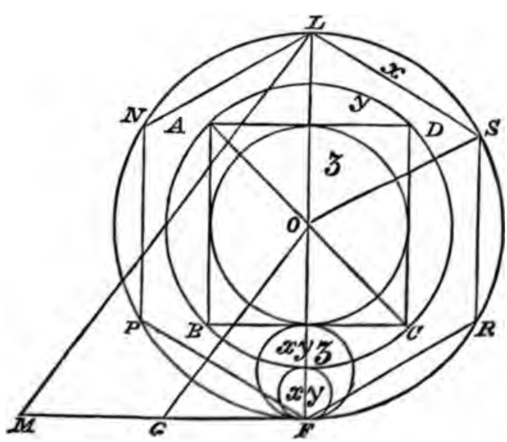
Dr. NEVINS referred to the variation, during long voyages, of the chronometer, as given by Mr. Hartnup, the Astronomer of our Observatory, and objected to Mr. Smith's argument, as based upon time by the chronometer and time by observation.

Mr. SMITH replied that Mr. Hartnup would tell Dr. Nevins that between the latitudes of 40° and 75° the chronometer could be adjusted so as not to vary at all. He argued that Mr. Jones must certainly have forgotten that all Astronomers agreed that the Sun had an onward motion, and consequently Mr. Jones's argument that the Earth moved in an ellipse was an absurdity. The Earth's motion must be spiral or cycloidal. If the subject of the discussion had been the motions of these bodies, he could have shown that the same harmony, beauty, and consistency prevailed in their motions, velocities, and comparative rates of speed, as he had endeavoured to shew to the meeting this evening, existed with regard to their dimensions and distances.

A cordial vote of thanks to Mr. Smith terminated the proceedings.

I exhibited the following Diagram to the Society, but it was impossible, in the time allotted to a Paper, to connect the subject with the properties of this geometrical figure.

*Diagram.*



I may, however, take this opportunity of calling the attenion of the Reader to the following facts, and leave him to pursue the inquiry further for himself. The right angled triangles L F M and O F G are similar triangles, and the sides containing the right angle are in the proportion of 4 to 3, by construction; therefore, the perimeter of the triangle L F M, and the perimeter of the inscribed regular hexagon LNPFRS to the circle X, are equal, and both = 6 (OF) the radius of the Circle X; therefore, if we suppose the circle X to represent the Earth, 10 times the perimeter of the right angled triangle L F M, or, 10 times the perimeter of the hexagon LNPFRS, = 60 times O F the radius of the circle X, = Moon's distance from the Earth; and these facts are confirmed by actual experiment, in ascertaining the Moon's distance by angles obtained from two distant points on the Earth's surface.* J. S.

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* "*The Solar System as It Is, and Not as It is Represented.*" By Captain R. J. Morrison, R.N.—In this Work the true theory of our Solar System is proclaimed to the world. It is true, that much remains to be done to fully elucidate the theory, and perfect it as a system, which involves the reconstruction of Mathematical Tables of Sines, Tangents, &c. But, the foundation truths of Astronomy are propounded by the Author, with a simplicity and clearness, that will baffle the intellect of man to controvert; and a careful study of this Work cannot fail to amply reward every sincere and earnest inquirer after Astronomical truth. Dr. Whewell, and the rest of the Astronomical fraternity, may sneer, and in their endeavours to "*guard the mysteries of their craft,*" they may attempt to make us believe that it would be dangerous to "*disturb existing systems;*" as if, forsooth, truth could be sacrificed to them and their "*vagaries.*" Vain men! They may have their day! But, in the long run, truth must and will prevail, and ultimately consign them and their complex systems to the shades of oblivion. (See also, *Astronomy in a Nut Shell,* by the same Author.)

# APPENDIX C.

## A FACT, OR NOT A FACT.

*From the Liverpool "Daily Post," September* 9, 1864.

THE following letter from Mr. JAMES SMITH puts the theory of "squaring the circle" to the test. Either the present Astronomers are wrong or he is wrong; but if he be right, they are certainly wrong, and of course his theory is of immense value :—

DEAR SIR,—From our conversation this afternoon in the Secretary's Office of the Exchange Room, I am induced to address a few observations to you, with reference to a practical application of the discovery of the true ratio of diameter to circumference in a circle.

For the purposes of Astronomical and Nautical science, the equatorial circumference of the Earth is divided into 360 degrees, and each degree is assumed to be equal to 60 nautical miles, equal to about 69½ English miles, and 360 x 60 = 21,600 is the Earth's equatorial circumference in nautical miles. This is made by Mathematicians, by calculations based on actual measurement, into 24,899 English miles of 1,760 yards each. The Earth's diameter they make about 7,912 miles. The ratio of 7,912 to 24,899 is not very far from the true proportions between the Earth's diameter and circumference, but sufficiently so to introduce an important error into all the nautical calculations based on this assumption.

Now, you know as well as I do, that the Great Creator of the Universe did not call the planetary bodies into existence, and arrange their dimensions, distances, and relations to each other, on a measure of value based on English miles; but in His wisdom He so created them that equatorial diameter is to equatorial circumference in the ratio o 8,000 to 25,000; and these are the proportions between the diameter

and circumference of all circular bodies, whether celestial or terrestrial, as may be demonstrated by pure geometry ; and, if these figures be taken to represent the Earth's dimensions, it matters not whether we call them miles or cubits, or by any other name, (it being sufficient for all practical purposes to know that a degree is,as nearly as possible, equivalent to 69½ English miles), time by observation and time by chronometer would agree on every part of the Earth's surface, if our Nautical tables were calculated upon this datum, and for the following reason :—Time is the measure of longitude, and is made so in all the Observatories of the world; and we divide time into days of twenty-four hours of sixty minutes each, which makes 24 x 60 = 1,440 minutes, the measure of a day of twenty-four hours, and this divided by 360° gives four minutes ; therefore, one degree of longitude is equal to four minutes of time. Every experienced mariner knows the law, that for every degree of longitude his ship makes as she sails, either eastward or westward, from any given point, the sun appears above the horizon four minutes sooner or later than at his point of departure, as indicated by chronometer. Hence in sailing round the globe, the mariner discovers, on returning to his point of departure, that he has lost or gained a day, according to the course he has taken on the voyage ; that is, whether westward or eastward ; and with this fact you will be quite familiar.

Now, my dear Sir, all existing Nautical tables are calculated on the theory that the Earth's circumference is less than 25,000 by a quantity represented by a little more than 100 ; and yet this apparently trifling difference produces an error in the latitudes between 40° and 50° (within which the track from this to New York may be said to be limited) of from 10 to 15 miles between time by observation and time by chronometer, which would be impossible if the parallels of latitude and meridians of longitude were based upon a true equatorial circumference of the Earth, and if Tables were calculated accordingly.

May I request the favour of your reading with this, what I have said on the subject in pages 7 and 8 of my paper, read before the Literary and Philosophical Society, on the dimensions and distances of the Sun, Moon, and Earth, a copy of which accompanies this communication, and which Captain Judkins, and many other nautical gentlemen of the highest intellect, have assured me is in exact conformity with their experience, but which they could never before explain.

I should have observed, that it is a remarkable fact that the Earth's circumference should be so very near 25,000 as measured in English miles; but this is as it were a matter of accident and cannot affect my argument, but makes the term "miles" not inappropriate.

I remain,

My dear Sir,

Yours very truly,

JAMES SMITH.

M. J. WHITTY, ESQ.

*Barkeley House, Seaforth, 7th Sept.,* 1864.

# APPENDIX D.

## ON THE DRUIDICAL REMAINS AT STANTON DREW, NEAR BATH.

*To the Editor of the "Times."*

SIR,—On the occasion of the late meeting of "The British Association for the Advancement of Science," I was one of a large party of its Members on a visit to the Druidical Remains at Stanton Drew, about eleven miles from Bath. These interesting relics of bygone times are deserving of notice, even as objects of mere curiosity, and cannot fail to afford an infinite amount of pleasure to Antiquaries; but, they appear to me to be deserving of further consideration and enquiry, viewed as records of the progress made in the Astronomical and Mathematical Sciences, at a period of time *now* blotted out from the page of written history. It strikes me there is strong evidence of a high order of design in the original construction of these Ancient Druidical Temples. But even if this view cannot be accepted, there are at any rate some very extraordinary coincidences with reference to them, to which I cannot resist the temptation to direct public attention, through the columns of your valuable Journal.

It was the first time the "British Association" had held its annual meeting at Bath, and on the occasion of the visit referred to, the proprietor of the property on which these interesting relics are located, had kindly bared a number of the prostrate pillars, which had long been buried beneath an amount of soil, that had produced many a crop for the sustenance of either man or beast.

These remarkable Druidical remains consist of 7 large isolated stones, and 3 circles. The smallest of these circles is 96 feet in diameter, and is composed of 8 stones. The intermediate circle is 120 feet in diameter, and is made up of 12 stones. The largest circle is composed of 36 stones, and is 300 feet in diameter. Some of these

stones are of great magnitude, one of which was fully exposed to view, and was estimated to weigh upwards of 23 tons.

With reference to the 7 isolated stones I may observe, that 7 is called the mystic number, and the decimal expression for unity divided by 7, (that is, the decimal expression for the fractional quantity $\frac{1}{7}$,) is ·142857 &c., and ·142857 is a recurring decimal. Now, if ·142857 &c. be multiplied by 2, 3, 4, 5, and 6, we obtain the decimal expressions of the fractional quantities $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$, which may be tabulated as follows:

$$\frac{1}{7} = \cdot142857 \text{ \&c.}$$
$$\frac{2}{7} = \cdot285714 \text{ \&c.}$$
$$\frac{3}{7} = \cdot428571 \text{ \&c.}$$
$$\frac{4}{7} = \cdot571428 \text{ \&c.}$$
$$\frac{5}{7} = \cdot714285 \text{ \&c.}$$
$$\frac{6}{7} = \cdot857142 \text{ \&c.}$$

The figures composing this Table add up to 2·999997, which divided by 3 gives ·999999, and is exactly equal to 7 times ·142857, therefore, $\frac{\cdot999999}{2\cdot999997}$ is an equivalent arithmetical expression for the ratio $\frac{1}{3}$; and 1 to 3 is the known and indisputable relation between the diameter of every circle, and the perimeter of its regular inscribed hexagon. Each row of figures in the Table adds up to 3 times 9, and it will be observed that whether the figures be taken vertically or horizontally, each row is composed of the same figures, with simply a change in the order of their arrangement.

Again: The number of stones by which the two smaller of these Druidical circles are indicated is 8 and 12, and the product of these two numbers is 96, and is equal to the diameter of the smallest circle. The largest circle is indicated by 36 stones, and $\frac{36 \times 8}{3}$ is also equal to 96, the diameter of the smallest circle. Hence, $(36 + 12) = \frac{36 \times 8}{6}$, and both equal 48, the radius of the smallest circle. The arithmetical mean between the diameter and radius of the smallest circle $= \frac{96 + 48}{2} = 72 =$ twice 36; and $36 - 12$, or $96 - 72 = 24$; and 24 is to 72 as the diameter of the circle to the perimeter of its inscribed regular hexagon. Now, we may conceive an equilateral triangle to be inscribed in the smallest circle, and we may further conceive that diameters of the circle might be so drawn, as to bisect the angles of the triangle and their opposite sides, and supposing this operation to have been performed, the diameter of the circle would be divided by the sides of the triangle into two parts, of which the value of the one

part would be 72, and the value of the other 24; and 72 and 24 are in the proportion of 3 to 1, that is, they are in the same proportion as the perimeter of any regular hexagon, and the diameter of its circumscribing circle.

Now, if the diameter of the smallest of these Druidical circles represent the perpendicular of a right-angled triangle, and $\frac{3}{4}$th parts of it base; that is, if the sides which contain the right angle in a right angled triangle be 96 and 72, the hypothenuse of the triangle = 120, and is exactly equal to the diameter of the intermediate Druidical circle. The diameter of the smallest circle is to 100 times unity in the proportion of 3 to 3·125, that is, 96 : 100 :: 3 : 3·125. Now, the fact is incontrovertible, that 3·125 times $96^2 = (96^2 + 72^2 + 120^2)$; therefore, 3·125 times the area of a square on the diameter of the smallest Druidical circle, is equal to the sum of the areas of the squares about a right-angled triangle, of which the sides containing the right angle are in the proportion of 3 to 4, and the longer of these sides equal to the diameter of the circle. But, 3·125 times $96^2$, or, $(96^2 + 72^2 + 120^2) = 28800$, and is equal to 100 times the perimeter of a regular inscribed hexagon to the smallest Druidical circle. For $\frac{96}{2} = 48 =$ radius of the circle, and 6 times radius = perimeter of an inscribed regular hexagon to every circle; therefore, $6 \times 48 = 288 =$ perimeter of an inscribed regular hexagon to the smallest of these Druidical circles; and 100 times 288 = 28800.

It may be objected that I have drawn a comparison between things that are unlike, since the one represents a line and the other a superficies; and that consequently no relation can possibly exist, or any comparison be made, between them; but this is a very frivolous objection, for the following reasons: If the diameter of a circle be 4, the value of the circumference and area are represented by the same arithmetical symbols; and if the circumference of a circle be 4, the values of the diameter and area are represented by the same arithmetical symbols. These are simply facts, which I defy any Geometer and Mathematician to controvert; and it would not be either more or less absurd to dispute *them*, than to raise a quibble as regards the facts to which I have called attention.

Again: The number of stones by which the smallest and largest of these Druidical circles are indicated, is 8 and 36, and the product of these two numbers is 288, and is equal to the perimeter of an inscribed regular hexagon to the smaller of these circles. The diame-

ter of the larger circle is 300 feet, and 288 is to 300 in the proportion of 3 to 3·125; therefore, the perimeter of a regular inscribed hexagon to the smaller circle, is to the diameter of the larger circle, in the proportion of 3 to 3·125. Now, 3·125 times 288 = 900, and is equal to the perimeter of an inscribed regular hexagon to the largest of the Druidical circles. For, the diameter of the largest circle is 300 feet, and 6 times radius = perimeter of a regular inscribed hexagon to every circle, therefore, 6 times $\frac{300}{2}$ = 900, = perimeter of an inscribed regular hexagon to the largest of the Druidical circles; and I beg to call the especial attention of Geometers and Mathematicians to the following incontrovertible facts:

If 900 represent the circumference of a circle, the area of this circle is to the area of an inscribed regular dodecagon to the largest of the Druidical circles, as the area of the dodecagon to the area of the largest Druidical circle; or in other words, if the perimeter of a regular inscribed hexagon to any given circle represent the circumference of another circle, the area of this circle is to the area of an inscribed regular dodecagon to the given circle, as the area of the dodecagon to the area of the given circle. This is either a fact, or not a fact, which any Mathematician may readily put to the test, whatever theory he may be pleased to adopt as to the true ratio of diameter to circumference in a circle.

For example: The diameters of the smallest and largest of the Druidical circles under consideration are 96 feet and 300 feet respectively, and the fact is incontrovertible, that 96 is to 300 in the proportion of 1 to 3·125. For the sake of argument let us assume that 1 to 3·125 is the ratio of diameter to circumference in a circle. Then, $\frac{300}{2}$ = 150 = radius of the largest Druidical circle, and 3·125 times $150^2$ = 70312·5, = area of the circle. 6 (radius x semi-radius) = area of an inscribed regular dodecagon to every circle, therefore, 6 (150 x 75) = 67500 = area of an inscribed regular dodecagon to the largest Druidical circle. 6 times radius = perimeter of a regular inscribed hexagon to every circle, therefore, 6 times 150 = 900, = perimeter of an inscribed regular hexagon to the largest Druidical circle. Now, if 900 represent the circumference of a circle, on our hypothesis, $\frac{900}{2(3·125)} = \frac{900}{6·25}$ 144 = radius of the circle; and 3·125 times $144^2$ = 64800 = area of the circle; and 64800 : 67500 :: 67500 : 70312·5. We may adopt any other ratio of diameter to circumference in a circle,

work out the calculations, and obtain similar results approximately; and we may make the approximations as close as we please; but, by no other ratio (within admissible limits) than that which exists between the diameter of the smallest and largest of these Druidical circles, can we ever arrive at exact Arithmetical results.

Again: Let the perimeter of a regular hexagon be 360. Then, $\frac{360}{3} = 120$ = diameter of a circumscribing circle to the hexagon, and is equal to the diameter of the intermediate Druidical circle. $\frac{120}{2} = 60$ = radius of the circumscribing circle to the hexagon, and the perimeter of the hexagon multiplied by the radius of the circumscribing circle, $= 360 \times 60 = 21600$ = the number of geographical miles into which the circumference of the Earth is divided for Astronomical and Nautical purposes. Now, 3·125 times 120 = 375, and 360 : 375 :: 3 : 3·125, which makes 120 circumferences of a circle exactly equal to 375 diameters. Therefore, 120 : 375 :: 8 : 25, and makes 8 circumferences of a circle exactly equal to 25 diameters. Therefore, $\frac{25}{8} = 3{\cdot}125$, which makes 1 to 3·125 the ratio of diameter to circumference in every circle, and this fact may readily be demonstrated by pure Geometry; and is in harmony with the proportions between the smallest and largest of the Druidical circles at Stanton Drew.

Can all the facts to which I have directed attention be mere coincidences? Do they not rather appear to be the result of design of the highest order? I confess, I cannot resist the inference, that the Ancients were better informed as to the ratio of diameter to circumference in a circle, than Mathematicians of the present day, notwithstanding the boasted intelligence of the age in which we live.

In conclusion I may observe, that in the "Times" of the 17th September, 1863, there appeared a long Letter from Mr. J. R. Hind, the well known Astronomer, announcing that Astronomers have hitherto been altogether in error as to the distance of the Earth from our great central luminary the Sun, making us some 4,000,000 miles nearer to the seat of light and heat than we were supposed to be. Now, the diameter of the intermediate circle in the interesting Druidical remains to which I have directed your attention is 120 feet, and when Astronomers become better informed as to the true relations existing between the planetary bodies of our solar system, they will discover that the Earth's mean distance from the Moon is equal to 120 times the Moon's diameter; the Sun's diameter equal to 120 times the Earth's diameter; and the Sun's mean distance from the

Earth equal to 120 times the Sun's diameter; and this explains how it happens, that to observers on the Earth's surface, the Sun and Moon are apparently bodies of equal magnitude, although the Sun's radius is actually equal to the diameter of the Moon's orbit.

I hope these few remarks may not be unacceptable to your numerous readers, and that they may induce further inquiry into this extremely interesting subject.

I remain, Sir,

Yours very respectfully,

JAMES SMITH.

Barkeley House, Seaforth,

Liverpool, 26th September, 1864.

ERRATA.

Page 21, lines 9 and 18 from bottom, *for* Wrottesly, *read* Wrottesley.